Simulating Electoral Systems: from open-list to single-member districts in Brazil

Gabriel C. Caseiro* and Leandro De Magalhães†
NYU *University of Bristol*

September 30, 2025

Abstract

Open-list electoral systems provide a complete rank ordering of all candidates-parties in a race covering a large area. Thus, we can observe a complete rank ordering of candidates-parties for *any* sub-area of that race. This allows us to simulate a single-member majoritarian electoral system based on revealed preferences with relative easy. Our benchmark is the 2022 election in Brazil. First, we simulate thousands of counterfactual intra-state constituency boundaries based on existing legal boundaries. Second, for each simulated constituency map we allocate a district winner using revealed preferences by four different methods. The individual-rank allocation produces results that have a similar ideological breadth of parties as the open-list system. The party-rank aggregation method delivers a two-party system. The left-right-rank aggregation produces complete dominance by right-wing parties. The left-center-right-rank aggregation does not identify a clear third party, suggesting instability.

*Email: gabrielcaseiro99@gmail.com
†Email: leandro.demagalhaes@bristol.ac.uk

1 Introduction

Comparing electoral systems is at the core of political science and political economy.[1] Proportional elections (PR) (compared to majoritarian) have been found to have higher government spending (Bawn and Rosenbluth (2003), Persson and Tabellini (2003)); less inequality (Crepaz (1998)); and more redistribution (Austen-Smith (2000) and Iversen and Soskice (2006)). Iversen and Soskice (2006) also show in a theoretical model that center-left parties are more likely to be in power under PR systems than under majoritarian systems. Similarly, Döring and Manow (2017) show that right-wing cabinets are more likely under majoritarian systems.

An important barrier in gaining insights from comparing electoral systems is that electoral systems at the country level do not vary at random, nor do they vary often. Most of the empirical evidence to support theoretical comparisons across electoral systems comes from cross-country studies,[2] or the study of rare electoral reform.[3]

Given the fundamental impact a change in political system may have on electoral and economic outcomes, it is not surprising that political debate often returns to electoral reform.

Brazil adopted proportional representation in the 1930s, followed by the majority of Latin America (Negretto and Visconti (2018)).[4] Currently, there are multiple bills in Congress – still at the committee stage – that propose electoral reform. One of them would implement in Brazil the single-member district electoral system currently used in the US, but with a similar population size per district within each state, and differing across states.

Unlike the US, where a whole-sale change in the electoral system has never been

[1] Rokkan (1970), Norris (1997), Boix (1999), Blais et al. (2005), Iversen and Soskice (2006); Persson and Tabellini (1999), Austen-Smith (2000), Milesi-Ferretti et al. (2002).

[2] Persson and Tabellini (2003), Iversen and Soskice (2006), Blume et al. (2009), Döring and Manow (2017), Gandhi et al. (2022).

[3] Norway 1919 (Cox et al. (2019) Fiva and Hix (2021), Paulsen (2022)); Switzerland 1918 (Emmenegger and Walter (2021)); Germany 1912 (Leemann and Mares (2014)); Russia 2005 (Gandhi et al. (2022)); Italy 2005 (Renwick et al. (2009), Viganò (2023)

[4] Before 1932 Brazilian states were split in electoral districts, each electing five representatives to the Lower House.

enacted, such institutional changes are often debated and implemented in other democracies. For example, electoral reform took place repeatedly in Italy in the last 30 years (Chiaramonte (2015)); Chile had a recent electoral reform in 2015 (Gamboa and Morales (2016)) and an attempt to adopt a new constitution in 2022; the UK had a referendum on electoral reform in 2011 (Renwick et al. (2009)), among others.

In this paper, we propose a novel approach to empirically evaluate and compare majoritarian and proportional electoral systems. We compare observed electoral results in an open-list proportional system with simulated results of a single-member district majoritarian system. This direction of analysis - from actual open-list to simulated single-member districts – can be constructed based on a purely data-driven approach. Open-list electoral systems provide a complete rank ordering of all candidates-parties in a race covering a large area. Thus, we can observe a complete rank ordering of candidates-parties for *any* sub-area, e.g., a polling station.[5] Each simulation builds up a district by joining small sub-areas - similarly to redistricting exercises for the US (e.g., McCartan and Imai (2023)).

If we were to use a closed-list proportional system as the basis for simulations, we would need to estimate voters's preferences regarding candidates for areas where those candidates were never present in the ballot box - only the party was. Merlo and Paula (2017) have shown this can be done with careful modelling, but it still requires the assumption that voters' preferences are ideological and can be represented in a multi-dimensional ideological space. The contribution of Merlo and Paula (2017) is to show that estimating such preferences is feasible with data for the 1999 European Parliament elections. They stop short, however, of simulating other electoral systems – which is the core of our paper.

Similar to us, Finan and Mazzocco (2021) simulate changes in electoral rules in Brazil, but only for the state of Roraima. They focus on two counterfactual reforms: approval voting and the imposition of a one-term limit, but maintain the electoral system as is: an open-list multi-member district. The complexity of their model means it can only be estimated for Brazilian states with a small number of members

[5]Open-lists electoral systems are common in the Americas (e.g., Chile, Peru, Ecuador, Colombia, Panama) and Europe (e.g., Germany, Italy, Finland, Poland, Ukraine), and also present in Asia (Indonesia, Sri Lanka and Japan).

of Congress, hence the choice of Roraima. In contrast to their work, our data driven approach and our objective to simulate a single-member majoritarian electoral system implies we can simulate elections for any state size. Moreover, our objective is to evaluate political outcomes such as the number of parties and ideological identity of those elected, not how public resources are allocated.

Iaryczower et al. (2022) estimate voters preferences in Brazil using the BLP (Berry et al. (1995)) estimation method from industrial economics. Each candidate for the Brazilian Lower House is a differentiated 'product' for which there are observable and unobservable characteristics. The vote share for each candidate is their market share. The method allows a parametrized version of voters' preferences to be estimated. The estimated preferences separately identifies the weights of ideology, observable characteristics, and unobservable valence. With the demand side estimated and fixed, Iaryczower et al. (2022) can simulate scenarios in which voters are able to pick their ideal candidate; and then compare welfare with the observed electoral system.

There are two fundamental differences between our paper and Iaryczower et al. (2022). First, the identification assumption. Our identifying assumption is that the complete ranking of candidates we observe at the precinct level is a sufficient statistic for a series of observables and non-observables determinants of highly localized preferences.[6] Their method relies on the functional form and parametrization of the voter's utility function, plus the choice of exclusion restrictions to identify changes in ideology that do not affect valence. Second, even though in theory their method could be used to estimate preferences at the highly localized level as we do, the practical estimate is likely to be unfeasible as they it requires observable characteristics at very fine cut geographical areas, which are not available.[7]

[6]At the extreme we would have each individual voter's complete rank. In this case, there would be no need to make assumptions about aggregation or the shape and parameters of voters' utility function.

[7]Computational burden may also be an issue, as Iaryczower et al. (2022) restrict their counterfactual simulations to the state of Bahia for that reason.

2 Simulation steps

2.1 Drawing single-member districts

Our first step is to simulate single-member electoral districts. This exercise is based on the literature that analyses the effects of redistricting and gerrymandering in the US using simulated redistricted maps. We implement the method proposed in McCartan and Imai (2023). This method imposes three desirable constraints: similar population per district, compactness, and preservation of administrative boundaries.

The important choice at this stage is the administrative boundaries to be used in the case of Brazil. We use the municipality boundaries and the boundaries of the electoral zones (*zonas eleitorais*). Electoral zones in Brazil are geographical areas within a state under the jurisdiction of an electoral notary who is responsible to coordinate voting in that area and under the supervision of electoral judges (*Tribunal Superior Eleitoral*). One municipality may include multiple electoral zones as is the case of the capital city of the state of São Paulo (see Figure 1), or one electoral zone may contain multiple municipalities and sometimes only some neighborhoods within it (see Figure 2).

Our results are robust to alternative unit to build districts: we create Voronoi polygons around each polling station,[8] and then use theses areas as the basic administrative units to build up to simulated districts. This alternative increases the numbers of available units to combine into districts. For example, in the state of São Paulo, there are 10,773 polling stations whereas there are 780 areas bounded by electoral zones or municipalities. Since results are similar, we use administrative boundaries as computationally less demanding.[9]

Using the Sequential Monte Carlo algorithm in McCartan and Imai (2023) we run 50,000 unique iterations of the algorithm for each state and set the number of districts to the number of seats currently allocated in the Brazilian Lower House to that state. For example, for the state of São Paulo we set as the objective 70 electoral

[8]Given a set of polling stations in a municipality space, a Voronoi polygon P covers all area in the municipality that is closest to P then any other polling station.

[9]Both methods can be compared for the state of São Paulo in the appendix, Tables 8 and 9.

districts, respecting the piece-wise boundaries of municipalities and electoral zones (Figure 3), compactness, and allowing for a maximum of 5% deviation in population size between districts. The algorithm is able to converge to a unique map in 1,102 instances for São Paulo. These are the 1,102 unique maps that we use to estimate counterfactual single-member district majoritarian electoral results and compare with the actual open-list results.

In Figure 4 we show an example of the simulated district boundaries (red) compared to the observed municipalities' and electoral zones' boundaries. In Figure 5 we show another example and include shading corresponding to how close voter population is to 500,000 within each simulated district (allowed to diverge by 5%).

Convergence of the algorithm is not guaranteed. In Figure 6 we color in red the states where the algorithm was able to converge, and in blue where the algorithm could not converge to the correct number of districts.

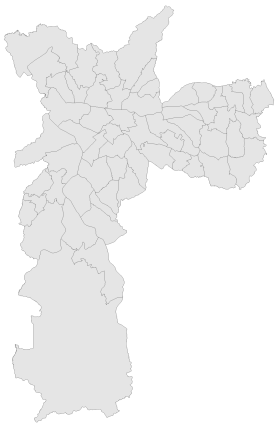


Figure 1: Municipality with multiple electoral Zones

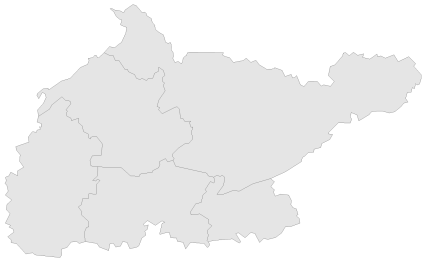


Figure 2: Zone with multiple municipalities

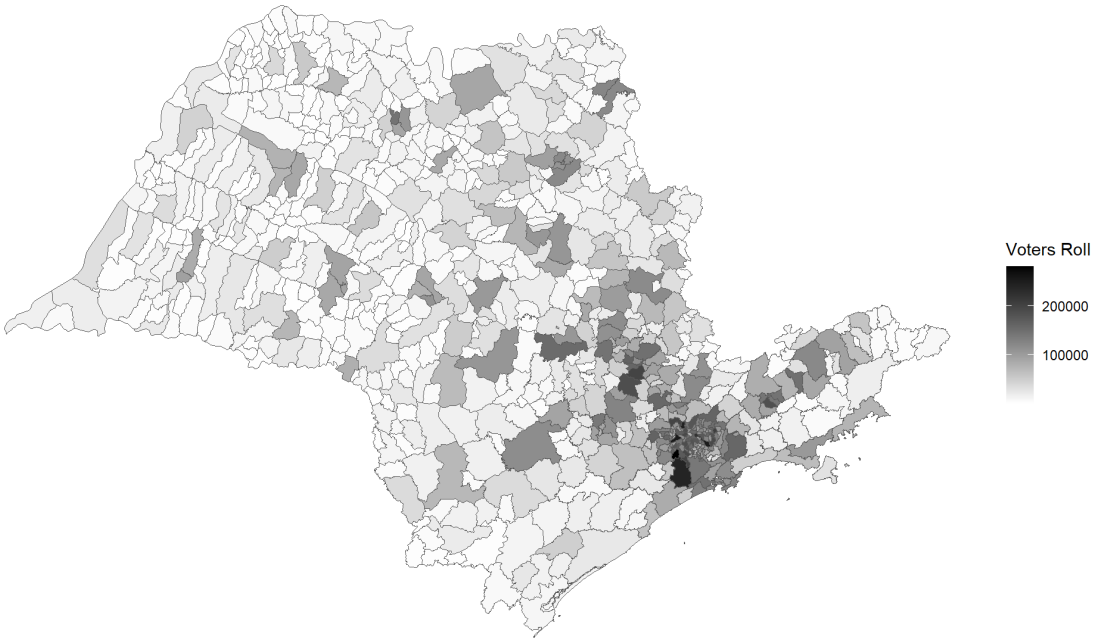


Figure 3: Voting Population within Municipal and Zone boundaries

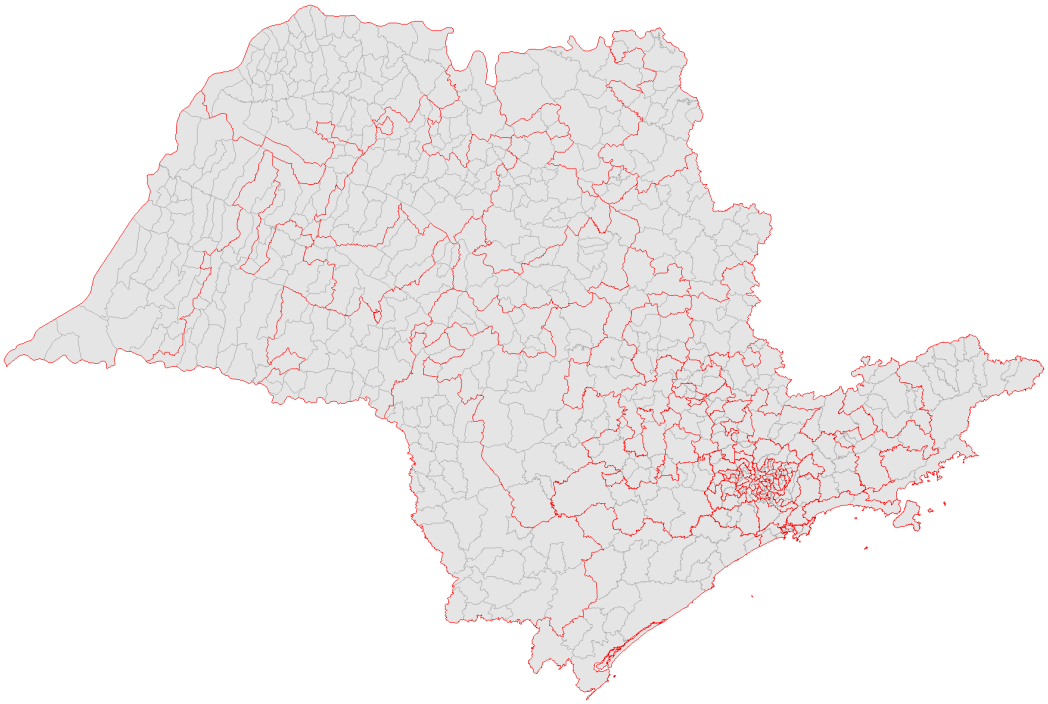


Figure 4: Example of Simulated boundaries

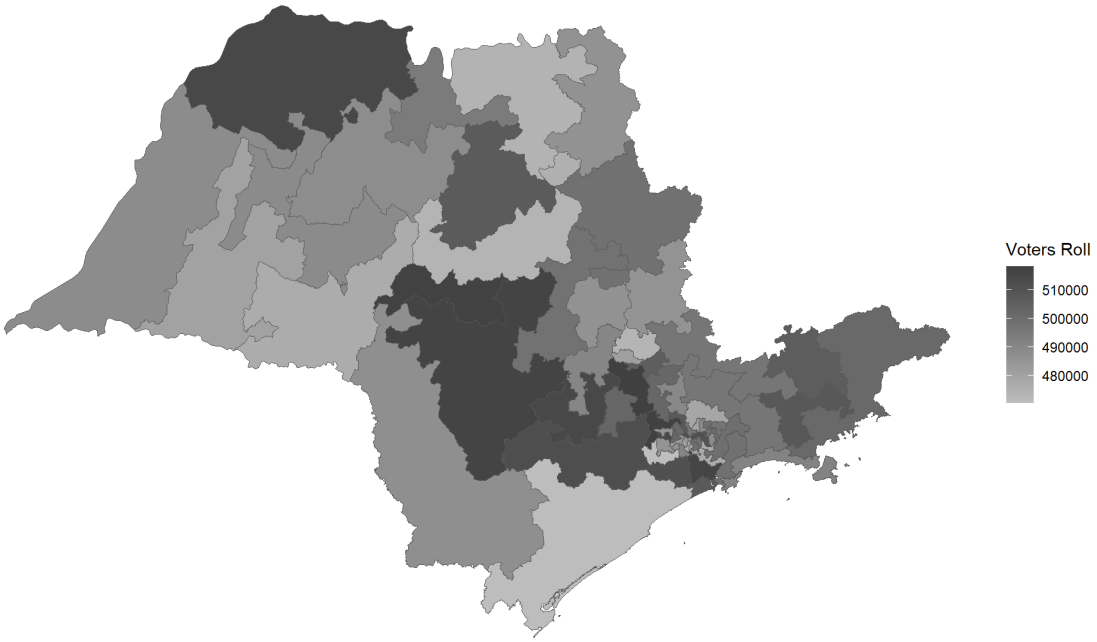


Figure 5: Example of Simulated boundaries - Population balance ($\leq 5\%$)

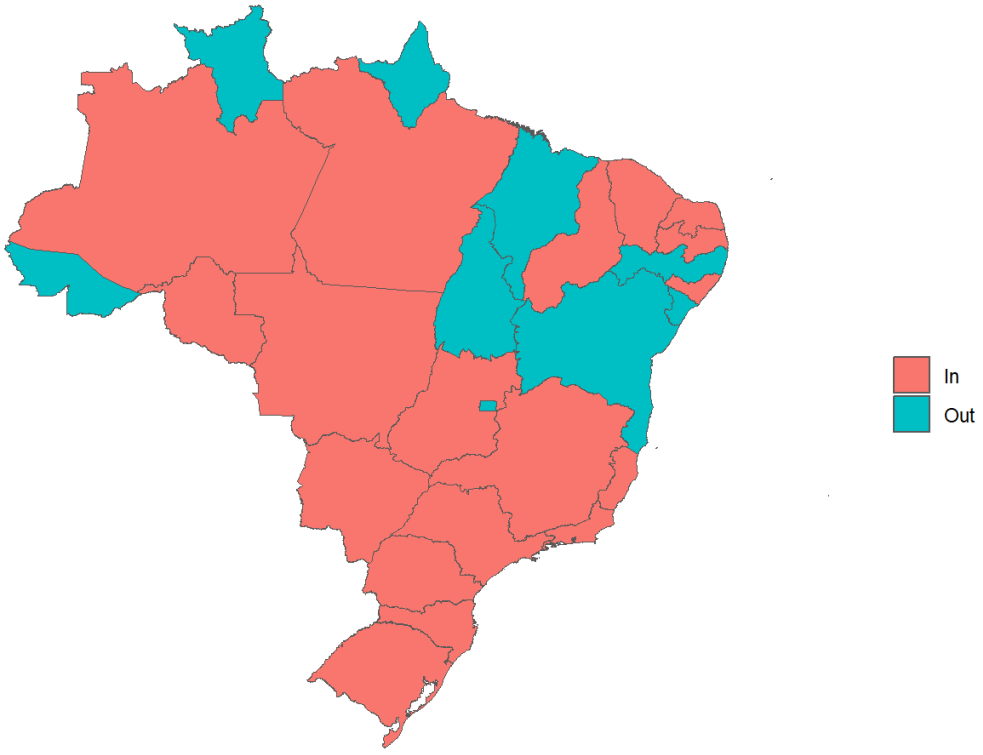


Figure 6: Simulated districts only available for Brazilian States in Red

2.2 Allocating candidates to districts

Our second step is to assign a winner for each of the simulated districts in each simulated map. We do so using the revealed complete ranking of candidates for each simulated district. We can do this because results are available by polling station. Thus, we can reconstruct a complete rank ordering of candidates for *any* geographical unit. Simulated districts are composed of multiple geographical units that respect both municipality and 'electoral zone' boundaries.

Importantly, every voter in each Brazilian state faces the exact same choice set of candidates and every candidate is clearly linked to a party.[10] Not only do they face the same choice set, voters also receive the same official campaign material. In Brazil, TV and radio time are allocated to candidates according to a predefined rule implemented equally across the entire state.

One concern that we believe to be mute in our case is that of selection into candidacy. The cost of running for office in Brazil is relatively low. Parties have an incentive to have multiple candidates with little or no chance of getting elected to boost the overall party vote.[11] Thus, we believe that the set of candidates observed in a given election is close to the universe of individuals willing to run for office at that point in time. In other words, our identifying assumption is that any candidate who would have run for office in a given single-member district majoritarian election would also have run for office in an open-list proportional election.

A remaining concern is that electoral systems differ not because they assign different candidates to office, but because they incentivize politicians to act differently. Indeed, Gagliarducci et al. (2011) use regression discontinuity design to show that similar politicians (the same average politician under a potential outcomes framework) elected under majoritarian or proportional rule, act differently according to how they were elected: through a majoritarian or proportional route. Our results

[10] To choose a candidate, the voter must enter the candidate's number. The first two digits are the 'party' number. The party also appears when the photo, name, and number of the chosen candidate appear for confirmation.

[11] The total party votes decides the number of seats. Once that is determined the party candidates are ranked by total votes.

only speak to how different electoral systems allocate candidates to office, not how they may behave differently given their method of election.

The intuition of the problem of assigning a candidate to a district is as follows. Very popular politicians are ranked first in more than one simulated district. In these cases, once this top politician is assigned to a particular district, we must use the complete rank order to inform us which candidate should be assigned to a district whose top choice is no longer available. The rank provides us with enough detail information to complete this task. Under each assumption, the district moves down their revealed rank in different ways depending on how we assume ideology of party identity plays a role. We allows this to vary in four different ways,

Independent of the preference aggregation method, the algorithm should have the following desirable features: i) the selected representative should reflect the electorate's relative preferences and ii) representatives should be assigned to the district with the highest possible support out of all other districts. The algorithm unfolds in several iterative steps as follows:

1. **Initial Preference Determination:** For each iteration, we first identify the candidate with the highest preference (i.e., the highest voting share) in every district that has not yet been assigned a representative.

2. **Iterative Process:** The algorithm repeats the above steps, recalculating the leading candidate for each remaining district and selecting the one with the highest share across these districts for assignment, until a representative is assigned to every district.

To illustrate how the algorithm works in practive, consider a scenario with three districts (D1, D2, and D3) and candidates (C1, C2, C3 and C4), where their voting shares in these districts are as follows:

- In D1, C1 leads with 50%, followed by C2 with 25%, C4 with 15%, and C3 with 10%.

- In D2, C1 again leads with 35%, but C2 is close behind with 32%, C4 has 20%, and C3 with 15%.

- In D3, C2 leads with 30%, C1 has 28%, C3, 22%, and C4 has 20%.

According to our algorithm:

1. C1, having the highest overall share, is elected in D1 in the first iteration.
2. With C1 now removed from consideration, C2, now the leading candidate with the highest share across the remaining districts, is elected in D2 in the subsequent iteration.
3. Finally, C3 is elected in D3, as she is the the leading remaining candidate in the last district.

This example clarifies how the algorithm addresses the issue of candidates who lead in multiple districts. The algorithm assigns individuals such as C2 to the district where their election has the highest vote support *based on the recalculated preferences after higher-ranked candidates in other districts are removed from the process.* C2 is the leading candidate in D3 initially. However, after C1 is elected in D1 and removed from further consideration, C2's strongest remaining support is in D2, not D3. This method prevents a scenario in which a candidate is elected in a district despite having a stronger voter support in another.

2.3 Individual-rank

First, we focus purely on the electoral preferences expressed over individual candidates through the open-list results, irrespective of party or ideological affiliation. The individual-rank algorithm considers the revealed preferences with the ordering of individual candidates in the simulated district. At the beginning of the algorithm, every candidate is considered running in every district. This initial step ensures that the selection process is grounded in the actual voting behavior of the electorate, with no candidate being excluded from consideration in any district. We discuss results by comparing the distribution of districts by party under the actual open-list electoral system and the simulated single-member district, but party is not used to allocated seats.

2.4 Party-rank

In this approach, we begin by assessing the performance of parties at the district level, aggregating the nominal votes received by each candidate of a party and the party's list votes. Once the leading party in a district is determined, we then apply the winning candidate selection algorithm. However, in this phase, the candidates considered for each district are exclusively those belonging to the district's preferred party. This means that the algorithm's focus narrows to selecting the most favored candidate within the context of the party that has garnered the highest overall support from the district's voters. This algorithm tells us what the simulated single-member district outcome is if voters have very strong party preferences and would use the party cue as the only cue in transferring their vote to a candidate down their preference rank.

2.5 Left-right-rank

Following a similar logic to the party-rank algorithm, the left-right-rank algorithm aggregates votes not based on parties but by ideological orientation. We assign parties to two broad categories: left and right. We use the left, right, and center-right classification in Bolognesi et al. (2022), which is based on interviews with political scientists.[12] Bolognesi et al. (2022)' classification of the main right-wing party, PL; the main left-wing party, PT; and the main center-right parties, MDB and PSDB match the classification using data on campaign contributions in Iaryczower et al. (2022).[13] In the lef-right-rank classification we assign every center-right party to the

[12]An alternative classification could be to use party support for gubernatorial candidates of other parties. In practice there would be few changes. In São Paulo (Table 8), for example, SOLIDARIEDADE moves from center-right to left and UNIAO from center-right to Right.

[13]Iaryczower et al. (2022) focus on the 2006, 2010, and 2014 election. Many parties have since disappeared or new ones have been created.

right.

Table 1: Ideological Preferences - Three Groups Division

Group	**Parties**
Left	PC do B, PCB, PCO, PDT, PSB, PSOL, PSTU, PT, REDE, UP
Center-Right	AVANTE, CIDADANIA, MDB, PMB, PMN, PODE, PROS, PSD, PSDB, PTB, PV, SOLIDARIEDADE
Right	AGIR, DC, NOVO, PATRIOTA, PL, PP, PRTB, PSC, REPUBLICANOS, UNIÃO

Within each district, after votes are aggregated to determine the dominant ideological preference (left or right), we apply the candidate selection algorithm. This algorithm tells us what the simulated single-member district outcome is if voters did not identify strongly with parties, but instead did identify strongly with two broad ideological camps. Voters would then use the left or right ideology cue as the only cue in transferring their vote to a candidate down their preference rank.

2.6 Left-center-right-rank

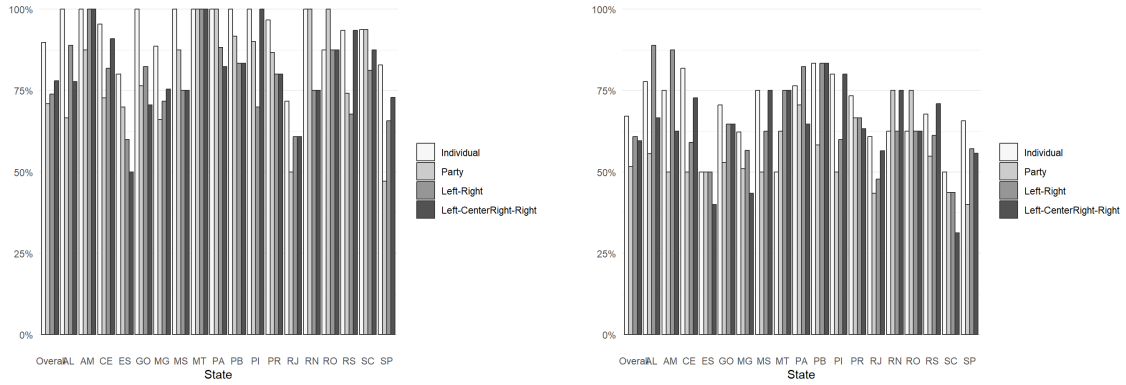
Finally, we allow three ideological groupings: left, center-right, and right. Votes are aggregated within each district according to these three ideological groups, and the algorithm proceeds to identify and assign the winning candidate from the group that garners the most support in each district.

3 Results

Our simulation of single-member districts and our allocation of one candidate to each district produces reasonable results. Figure 7a shows that, overall approximately 90% candidates that were actually elected in the open-list system are also elected in at least one simulation of the single-member district electoral system when candidates

are allocated to districts according to the individual-rank algorithm. Numbers go down to approximately 75% for the other three allocation methods. There is variation across states, but in no state and method does this number go below 50%. In Figure 7b we show that overall, between 50% and 70% of those actually elected in the open-list system are elected in at least 50% of the simulations; with varying but consistent numbers across states.

Figure 7: Outcomes of actually elected in simulations



(a) % elected at least once

(b) % elected more than 50%

The above numbers show that our simulations are mostly allocating successful politicians across districts, and to a lesser extent, substituting actually elected politicians with non-elected ones. Both figures also show that the individual-rank allocation algorithm is the closest to the actual open-list electoral system and this holds for almost every state.

The main summary of our results is Table 2. We list all the parties that took part in the 2022 Congressional election and indicate their ideological classification: blue for 'right', light-blue for 'centre-right' and red for 'left' (red is the color of PT and blue is the color of PL). In column 1 we report the number of representatives (*deputados federais*) elected by each party in the actual observed 2022 open-list election. The two main parties are the right-wing PL with 80 elected representatives and the left-wing PT with 58. Overall, there were 146 representatives elected by right-wing parties, 143 by center-right parties, and 94 elected by left-wing parties. Out of a total of 383

seats in our simulations.[14]

[14]In total the Brazilian Lower House has 513 seats, but our district construction algorithm did not converge in for all states.

Table 2: Observed Open-list Proportional vs Simulated Single-Member Majoritarian

			Simulated			
Party	**Actual**	**Plurality**	**Individual**	**Party**	**Left-Right**	**Left-CR-Right**
PL	80	70	58	172	68	85
PT	58	56	47	54	1	17
UNIÃO	40	30	44	28	55	63
MDB	34	35	42	28	47	26
PSD	32	34	31	21	43	21
PP	31	34	35	25	44	50
REPUBLICANOS	22	26	20	11	27	39
PSDB	12	13	13	7	18	11
PSOL	12	12	10	6	0	5
PDT	11	11	12	11	1	9
PODE	10	10	15	6	21	4
PSB	7	7	10	1	0	2
AVANTE	5	4	7	0	8	1
CIDADANIA	5	6	8	5	9	7
PSC	5	7	9	3	10	13
NOVO	3	4	2	1	4	7
PATRIOTA	3	3	5	1	5	13
PROS	3	4	2	0	5	2
PV	3	3	4	1	5	2
SOLIDARIEDADE	3	6	5	2	9	2
PC do B	2	3	1	0	0	1
PTB	1	4	0	0	1	1
REDE	1	1	1	0	0	0
AGIR	0	0	0	0	0	1
DC	0	0	1	0	1	1
PMN	0	0	1	0	1	0
Total Right	184	174	174	241	214	272
Total Center-Right	108	119	128	70	167	77
Total Left	91	90	81	72	2	34

Note: Parties are allocated either as 'left' (red) or right' (blue) according to Bolognesi et al. (2022). Column 1 presents the number of representatives elected by party in the actual open-list system. In column 2, 3, 4 and 5 we present the 70 candidates elected in the most number of the 1,102 simulated constituency maps; column 2 according to the rank order observed in the open-list vote; column 3 votes are first aggregated by party within each simulated district; column 4 votes are aggregate by 'left' or 'right' within each simulated district; ; column 4 votes are aggregate by 'left' or 'right' in three groups within each simulated district.

In column 2 we report the outcome for the individual-rank non-ideological algorithm. We list the party affiliation of the individuals that were elected more often in the thousands of generated constituency maps to compare it with the actual disribution. The similarity between column 1 and 2 are striking. The order of the first four parties is maintained, and only small deviations in the ordering are observed lower down the ranking. The only notable difference is the reduction in the number of PL representatives by 30% and of PT representatives by 15%.

The similarity between columns 1 and 2 is possibly the best sanity check for our simulation exercise. The algorithm allocates one candidate to each district using the district-specific full rank of individuals. Those selected by the algorithm in each district have a very low individual vote share – defined as the vote share after aggregating the smaller geographical units into the simulated districts. In Figure 8 in the appendix we can see that the individual vote share is on average 10% of the district vote. This means that the algorithm and the district-specific ranking are doing a lot of work to allocate individual candidates to particular districts (as opposed to certain individuals having high vote shares in the simulated district and being an 'obvious' choice). Nevertheless, the individual-rank non-ideological simulation delivers a surprisingly similar outcome to the actual observed open-list.

Another insight of the comparison between columns 1 and 2 is that removing ideology from the allocation of seats entirely, has but a small effect on the reduction of ideological polarization. The overall number of seats allocated to the Right and Left reduces by 16% and 10% respectively and the Center-right has a gain of 23%. Most of the Right and Left losses come from a reduction in the seats allocated to the two main parties, the PL, and the PT. These is due to both parties being those that benefit the most from their large party votes, usually driven by super-star candidates. These large party votes allow the PL and PT to elected candidates with low individual vote shares.[15] This suggests that our individual-rank simulation of single-member districts would yield a very similar result to a pure individual open-list system (without the party vote count).

[15] As a reminder the open-list electoral system in Brazil first counts the vote share by party-coalition, which decided the total number of seats that that party-coalision; then the intra-party rank determines who is elected.

A summary of the comparison between column 1 and 2 and Figures 7a and 7b reads as follows. textbfFor fixed individual-rank preferences as observed in the data, there is little difference in outcomes, i.e., who and what parties get elected, between an open-list system and a single-member-district elected by first-past-the-post method.

The single-member district simulations start to deliver clearly different results once we allow preferences to be transferred not to the next name down the list, but the name down the list from the same party or ideology.

In column 3 we report the outcome for the party-rank algorithm. The simulated single-member majoritarian elections clearly departs form the actual open-list results. Despite being based on the same revealed individual-rank preferences. The main result here is that once we aggregate votes by party we move decisively towards a two-party system. This move is not mechanical. In the presence of a third or fourth party which is able to consistently achieve the highest vote share across many districts, the two-party result would not obtain. If different pairs of parties disputed the dominance in different states, also the two-party result would not obtain.

In practice, the right-wing party PL wins in 172 districts and the left-wing PT wins in 54 districts. The right or center-right party with the most districts after the PL has 28 votes, i.e., 16% of the number of districts won by the PL. Similarly, the left party with the most districts after the PT has 20% of the districts gained by the PT. In the state of São Paulo the two-party results is even stronger, only three districts out of São Paulo's 70 are allocated to a party other than PL or PT (Table 8 in the appendix).

No center-right party emerges as a focal point for a possible third party system. The three top center-right parties have 28, 28, and 25 districts each. The sum of all three parties is not half of the districts won by the leading right party, the PL.

In column 4 we report the outcomes for the left-right-rank algorithm. This simulation imposes two ideological grouping, left and right. Note that even though we separately present the count for right and center-right, in allocating seats we ignore this distinction. The PL is still the leading party on the right, with 68 seats, but

other parties are not far behind: UNIÃO with 55 seats and MDB with 47 seats. The main result in column 4, however, is that the entirety of the left leaning parties in all states, and including the PT, are wiped out. All but seven individuals from the left are able to be allocated to a district, once we pool together the left vote. The main leftist party, the PT, is only able to elected one member of Parliament. The left wipe-out is also visible in the state of São Paulo, where no candidate from the left is able to win a seat in the simulated districts (Table 8 in the appendix).

The above result is surprising. In the 2022 election the PT candidate, Lula, won 48% of the national vote in the first-round and 51% in the second round. Despite this, the left was unable to gather a majority of votes in virtually any simulated district. One counterfactual is to assign the district to the left or right by the first-round of the presidential vote. We could then compare the distance from those preferences to that shown by the right-left aggregation. Moreover, we show that the left-leaning vote is geographically disperse.

In column 5, the introduction of a center-right category restores the ability of the left to be elected in some districts, 36 (approximately one-third of the number of actual left candidates elected through the open-list system). The PL regains its prominent role in the right with 85 seats, with the following party classified as Right having less than half its seats. The center-right again fails to produce a clear dominant party; UNIÃO has 63 seats, and PP has 50 seats. The main insight gained from this exercise is that no single party seems to dominate in the 'center-right'. This lack of a clear latent dominant party in the center-right suggests a three-party outcome would be unstable or difficult to establish. This results also hold in São Paulo state (Table 8 in the appendix).

Importantly, no matter which algorithm we use to allocate seats, there is a clear reduction in the number of members of parliament from the left. The largest reduction occur when we aggregate preferences by two or three ideological groupings. This also holds at the state level as can be seen for São Paulo (Table 8 in the appendix). **In summary, a change in the Brazilian electoral system to a single-member district would be detrimental to left.**

In Table 3 we can explore another hypothesis regarding the effects of the adoption of single-member districts: the effect on the number of parties. Persson et al. (2003)

and Fiva and Hix (2021) suggest that single-member districts are likely to produce lower number of parties than open-list. This is not clear in from our results, however. The overall average number of parties – compared to the actual number under open-list – increases if the allocation is by individual-rank or right-left-rank; but decreases with the party-rank and with the left-center-right-rank.

This suggests that if we fix preferences and simulate single-member districts, the number of parties does not necessarily go down. A reduction of the number of parties, potentially to two or three may require some endogenous choices by parties and voters, e.g., the merging of parties and/or strategic voting.

Our results suggest that an eventual reform could lead to a two-party system. In column 3 of Table 3 we can see that if preferences were purely driven by party-rank, then the adoption of single-member districts would immediately create a two/three party system in all but a few states. This would happen without the need for the merger of parties or strategic voting. There is a potential for a 'latent' two-party system, specially given the dominance by the PL and the PT. Furthermore, there is room but no clear 'candidate' for a third party in the center. The left-center-right rank algorithm delivers an average of close to five parties per state. But as seen in Table 2, column 5, there is no clear focal party in the center. A focal center party would have to emerge dynamically, with mergers, or the center would be incorporated by the right.

Table 3: Effective Number of Parliamentary Parties (ENPP)

State	Actual Open-List	Individual	Party	Left-Right	Left-CenterRight-Right
AL	3.52	3.86	1.25	3.52	2.61
AM	4.57	6.40	2.67	5.33	5.33
CE	5.63	5.76	2.60	4.48	4.48
ES	5.56	6.25	5.56	5.56	4.17
GO	7.81	8.76	3.85	7.05	5.67
MG	8.97	11.75	2.45	11.02	8.54
MS	3.56	4.57	1.60	3.56	2.91
MT	2.67	4.57	2.13	3.56	3.56
PA	2.92	3.57	1.41	3.48	1.86
PB	6.00	5.54	2.88	4.80	4.80
PI	3.33	2.94	1.22	2.94	3.33
PR	7.38	8.49	4.74	6.72	8.49
RJ	8.40	7.15	1.92	6.34	4.68
RN	2.67	4.00	3.20	2.91	2.46
RO	2.67	4.00	2.29	4.00	4.00
RS	10.12	10.33	4.12	8.36	10.12
SC	4.57	7.53	1.13	7.53	4.57
SP	8.51	11.09	1.56	10.17	6.19
Average	5.48	6.69	2.59	5.58	4.78

Table 4: Candidates Characteristics

			Simulated			
	Actual	**Plurality**	**Individual**	**Party**	**Left-Right**	**Left-CR-Right**
Age	50.25	50.31	49.95	50.06	49.62	50.30
Sex - Male (%)	81.98	82.25	83.55	77.28	88.25	84.33
Color - White (%)	77.55	77.28	80.16	77.81	79.37	77.81
Married (%)	68.93	66.84	69.19	63.97	72.58	70.76
Completed College (%)	82.77	83.03	82.51	79.11	78.85	78.85
Born in same state (%)	82.51	82.77	82.51	79.63	83.29	80.94
Reelection (%)	56.92	59.53	43.60	33.94	40.73	42.04
Elected (%)	100.00	86.95	67.36	52.48	60.57	59.53

4 Aggregating localized preferences

In this section we use information from the very localized ranks at the polling station level not only to rank candidates, but to determine how each geographical unit would transfer their choice down the ranking. We allow one polling station to pass it through the party-rank, another through the individual-rank and so on.

Table 5: Observed Open-list Proportional vs Simulated Single-Member Majoritarian (Intra-Secao Redistribution)

		Simulated	
Party	**Actual**	**Party**	**Left-Right**
PL	80	112	58
PT	58	43	21
UNIÃO	40	31	51
MDB	34	40	45
PSD	32	33	39
PP	31	30	37
REPUBLICANOS	22	12	24
PSDB	12	15	19
PSOL	12	14	4
PDT	11	16	9
PODE	10	8	14
PSB	7	4	5
AVANTE	5	3	9
CIDADANIA	5	7	8
PSC	5	6	11
NOVO	3	2	3
PATRIOTA	3	1	6
PROS	3	1	5
PV	3	1	4
SOLIDARIEDADE	3	3	7
PC do B	2	0	1
PTB	1	0	1
REDE	1	0	1
DC	0	1	1
Total Right	184	195	191
Total Center-Right	108	111	151
Total Left	91	77	41

Note: Parties are allocated either as 'left' (red) or right' (blue) according to Bolognesi et al. (2022).

Table 6: Candidates Characteristics
(Intra-Secao Redistribution)

		Simulated	
	Actual	**Party**	**Left-Right**
Age	50.25	50.09	49.84
Sex - Male (%)	81.98	79.37	85.38
Color - White (%)	77.55	80.16	81.72
Married (%)	68.93	68.41	68.41
Completed College (%)	82.77	82.77	82.25
Born in same state (%)	82.51	81.46	84.07
Reelection (%)	56.92	43.34	41.78
Elected (%)	100.00	63.19	62.66

5 Discussion

We find surprising that the open-list proportional system delivers similar distribution of parties as the simulated single-member district majoritarian system, in which voters simply rank individuals and ignore party affiliation altogether. This suggests that parties are not used to inform voting choices in the Brazilian open-list electoral system. Such an interpretation would be in-keeping with a vast literature that describes Brazil as having a weak party system (e.g., Klašnja and Titiunik (2017)) and the voting being focused on individual candidates (De Magalhaes (2015)).

Another surprising result is the easy with which we obtain a two-party systems once we aggregate votes by party. We had no clear prior that one of two parties would dominate in every district. But this is what we find. This result supports a common view that single-member majoritarian system may be predisposed to a two-party system - or less party fragmentation, at least - as argued by Persson et al. (2003) and Fiva and Hix (2021), and as is the case in the US and the UK. This result also makes the third simulation more plausible, as it would be likely that after a reform, politician would migrate to either of the two clearly dominating parties; potentially creating a two-party system. Even when we allow for a third center-right ideological grouping, no single party dominates the center.

All four simulations deliver a higher number of right-wing representatives than the observed results in the open-list election. This veering to the right is complete in the third simulation, in which we impose a two-party system; all elected representatives are from the right and the left elects no one. This result supports the theoretical argument proposed by Iversen and Soskice (2006) that center-left parties are more likely to be in power under PR systems than under majoritarian systems; and the results regarding more right-wing cabinets under majoritarian systems (Döring and Manow (2017)).

References

Austen-Smith, D. (2000). Redistributing income under proportional representation. *Journal of Political Economy*, 108(6):1235–1269.

Bawn, K. and Rosenbluth, F. M. (2003). Coalition parties versus coalitions of parties: How electoral agency shapes the political logic of costs and benefits. *Available at SSRN 1154118*.

Berry, S., Levinsohn, J., and Pakes, A. (1995). Automobile prices in market equilibrium. *Econometrica*, 63(4):841–890.

Blais, A., Dobrzynska, A., and Indridason, I. H. (2005). To adopt or not to adopt proportional representation: The politics of institutional choice. *British journal of political science*, 35(1):182–190.

Blume, L., Müller, J., Voigt, S., and Wolf, C. (2009). The economic effects of constitutions: replicating—and extending—Persson and Tabellini. *Public Choice*, 139:197–225.

Boix, C. (1999). Setting the rules of the game: the choice of electoral systems in advanced democracies. *American political science review*, 93(3):609–624.

Bolognesi, B., Ribeiro, E., and Codato, A. (2022). Uma nova classificação ideológica dos partidos políticos brasileiros. *Dados*, 66:e20210164.

Chiaramonte, A. (2015). The unfinished story of electoral reforms in Italy. *Contemporary Italian Politics*, 7(1):10–26.

Cox, G. W., Fiva, J. H., and Smith, D. M. (2019). Parties, legislators, and the origins of proportional representation. *Comparative Political Studies*, 52(1):102–133.

Crepaz, M. M. (1998). Inclusion versus exclusion: Political institutions and welfare expenditures. *Comparative politics*, pages 61–80.

De Magalhaes, L. (2015). Incumbency effects in a comparative perspective: Evidence from Brazilian mayoral elections. *Political Analysis*, 23(1):113–126.

Döring, H. and Manow, P. (2017). Is proportional representation more favourable to the left? Electoral rules and their impact on elections, parliaments and the formation of cabinets. *British Journal of Political Science*, 47(1):149–164.

Emmenegger, P. and Walter, A. (2021). Disproportional threat: Redistricting as an alternative to proportional representation. *The Journal of Politics*, 83(3):917–933.

Finan, F. and Mazzocco, M. (2021). Electoral incentives and the allocation of public funds. *Journal of the European Economic Association*, 19(5):2467–2512.

Fiva, J. H. and Hix, S. (2021). Electoral reform and strategic coordination. *British Journal of Political Science*, 51(4):1782–1791.

Gagliarducci, S., Nannicini, T., and Naticchioni, P. (2011). Electoral rules and politicians' behavior: a micro test. *American Economic Journal: Economic Policy*, 3(3):144–174.

Gamboa, R. and Morales, M. (2016). Chile's 2015 electoral reform: changing the rules of the game. *Latin American Politics and Society*, 58(4):126–144.

Gandhi, J., Heller, A. L., and Reuter, O. J. (2022). Shoring up power: Electoral reform and the consolidation of authoritarian rule. *Electoral Studies*, 78:102475.

Iaryczower, M., Montero, S., and Kim, G. (2022). Representation failure. Technical report, National Bureau of Economic Research.

Iversen, T. and Soskice, D. (2006). Electoral institutions and the politics of coalitions: Why some democracies redistribute more than others. *American political science review*, 100(2):165–181.

Klašnja, M. and Titiunik, R. (2017). The incumbency curse: Weak parties, term limits, and unfulfilled accountability. *American Political Science Review*, 111(1):129–148.

Leemann, L. and Mares, I. (2014). The adoption of proportional representation. *The Journal of Politics*, 76(2):461–478.

McCartan, C. and Imai, K. (2023). Sequential Monte Carlo for sampling balanced and compact redistricting plans. *The Annals of Applied Statistics*, 17(4):3300–3323.

Merlo, A. and Paula, Á. d. (2017). Identification and estimation of preference distributions when voters are ideological. *The Review of Economic Studies*, 84(3):1238–1263.

Milesi-Ferretti, G. M., Perotti, R., and Rostagno, M. (2002). Electoral systems and public spending. *The quarterly journal of economics*, 117(2):609–657.

Negretto, G. L. and Visconti, G. (2018). Electoral reform under limited party competition: The adoption of proportional representation in latin america. *Latin American Politics and Society*, 60(1):27–51.

Norris, P. (1997). Choosing electoral systems: proportional, majoritarian and mixed systems. *International political science review*, 18(3):297–312.

Paulsen, T. (2022). Does proportional representation increase redistribution? Evidence from early 20th century Norwegian municipalities. *Electoral Studies*, 78:102494.

Persson, T., Roland, G., and Tabellini, G. (2003). How do electoral rules shape party structures, government coalitions, and economic policies?

Persson, T. and Tabellini, G. (1999). The size and scope of government:: Comparative politics with rational politicians. *European economic review*, 43(4-6):699–735.

Persson, T. and Tabellini, G. (2003). The economic effects of constitutions: what do the data say.

Renwick, A., Hanretty, C., and Hine, D. (2009). Partisan self-interest and electoral reform: The new Italian electoral law of 2005. *Electoral studies*, 28(3):437–447.

Rokkan, S. (1970). *Citizens ElectionsParties*. NewYork: David McKay.

Viganò, E. A. (2023). Electoral Incentives and Geographical Representation: Evidence from an Italian Electoral Reform. *Legislative Studies Quarterly*.

A Appendix

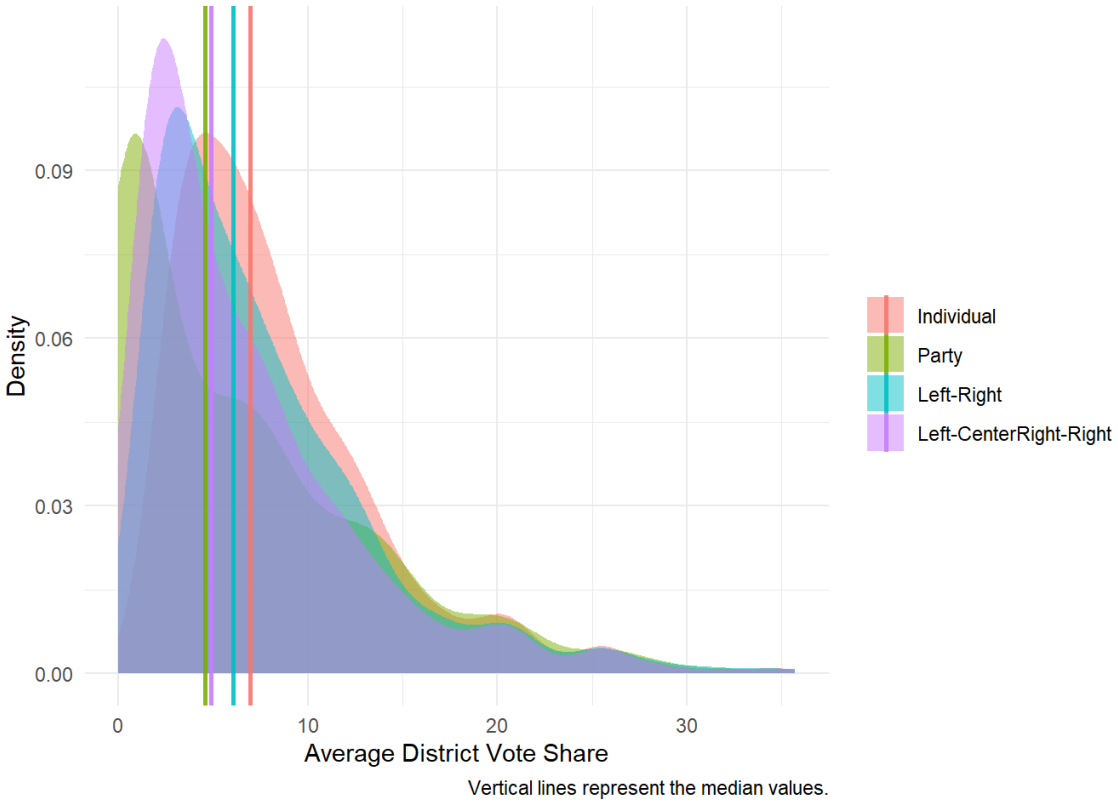


Figure 8: Average simulated-district vote shares for candidates elected at least once in simulations

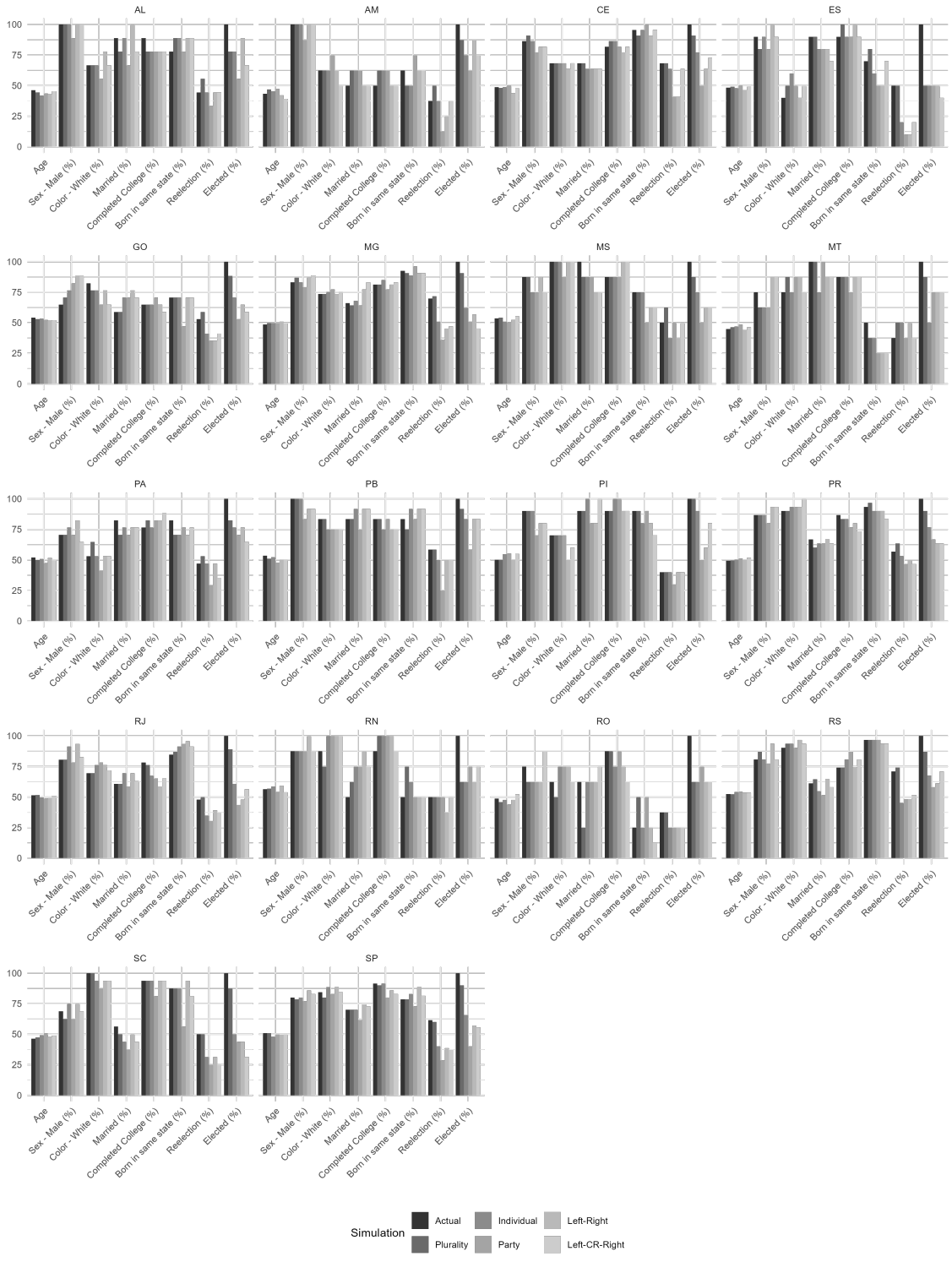


Figure 9: Candidates Descriptive Stats by State

Table 7: Observed Open-list Proportional vs Simulated Single-Member Majoritarian Changes

			Simulated			
Party	**Actual**	**Plurality**	**Individual**	**Party**	**Left-Right**	**Left-CR-Right**
PL	80	-10	-22	+92	-12	+5
PT	58	-2	-11	-4	-57	-41
UNIÃO	40	-10	+4	-12	+15	+23
MDB	34	+1	+8	-6	+13	-8
PSD	32	+2	-1	-11	+11	-11
PP	31	+3	+4	-6	+13	+19
REPUBLICANOS	22	+4	-2	-11	+5	+17
PSDB	12	+1	+1	-5	+6	-1
PSOL	12	0	-2	-6	-12	-7
PDT	11	0	+1	0	-10	-2
PODE	10	0	+5	-4	+11	-6
PSB	7	0	+3	-6	-7	-5
AVANTE	5	-1	+2	-5	+3	-4
CIDADANIA	5	+1	+3	0	+4	+2
PSC	5	+2	+4	-2	+5	+8
NOVO	3	+1	-1	-2	+1	+4
PATRIOTA	3	0	+2	-2	+2	+10
PROS	3	+1	-1	-3	+2	-1
PV	3	0	+1	-2	+2	-1
SOLIDARIEDADE	3	+3	+2	-1	+6	-1
PC do B	2	+1	-1	-2	-2	-1
PTB	1	+3	-1	-1	0	0
REDE	1	0	0	-1	-1	-1
AGIR	0	0	0	0	0	+1
DC	0	0	+1	0	+1	+1
PMN	0	0	+1	0	+1	0
Total Right	184	-10	-10	+57	+30	+88
Total Center-Right	108	+11	+20	-38	+59	-31
Total Left	91	-1	-10	-19	-89	-57

Note: Parties are allocated either as 'left' (red) or right' (blue) according to Bolognesi et al. (2022). Column 1 presents the number of representatives elected by party in the actual open-list system. In column 2, 3, 4 and 5 we present the 70 candidates elected in the most number of the 1,102 simulated constituency maps; column 2 according to the rank order observed in the open-list vote; column 3 votes are first aggregated by party within each simulated district; column 4 votes are aggregate by 'left' or 'right' within each simulated district; ; column 4 votes are aggregate by 'left' or 'right' in three groups within each simulated district.

Table 8: São Paulo: Observed Open-list Proportional vs Simulated Single-Member Majoritarian

	Number of elected by party and ideology				
	Actual	Simulated			
Party	Open-list	Individual rank	Party Agg.	Left-Right Agg.	Three Party Agg.
PL	17	9	52	12	16
PT	11	9	15	0	11
UNIÃO	6	8	0	11	3
MDB	5	6	0	7	4
PSOL	5	4	2	0	4
REPUBLICANOS	5	5	1	8	11
PP	4	4	0	5	2
PODE	3	5	0	7	1
PSD	3	5	0	5	7
PSDB	3	7	0	8	1
CIDADANIA	2	1	0	2	1
PSB	2	3	0	0	2
NOVO	1	0	0	1	0
PSC	1	0	0	1	4
REDE	1	0	0	0	0
SOLIDARIEDADE	1	2	0	0	1
PATRIOTA	0	1	0	1	0
PTB	0	1	0	1	1
AVANTE	0	0	0	1	1
Total Right	37	32	53	44	43
Total CenterRight	14	22	0	26	10
Total Left	19	16	17	0	17

Note: Parties are allocated either as 'left' (red), 'right' (cyan) or 'center-right' (cyan) according to Bolognesi et al. (2022). Column 1 presents the number of representatives elected by party in the actual open-list system. Columns 2 to 5 show the 70 candidates elected in the most number of the 1,102 simulated constituency maps: column 2 ranks candidates by open-list vote; column 3 aggregates by party in each simulated district; column 4 aggregates by left/right; and column 5 aggregates into three ideological blocks.

Figure 10: Polling station borders

Table 9: São Paulo - Polling station: Observed Open-list Proportional vs Simulated Single-Member Majoritarian

	Number of elected by party and ideology				
	Actual	Simulated			
Party	Open-list	Individual rank	Party Agg.	Left-Right Agg.	Three Party Agg.
PL	17	7	52	11	21
PT	11	10	14	0	4
UNIÃO	6	7	0	9	12
MDB	5	7	1	9	0
PSOL	5	4	1	0	1
REPUBLICANOS	5	5	1	6	13
PP	4	3	0	5	9
PODE	3	6	0	7	0
PSD	3	7	1	8	0
PSDB	3	5	0	7	0
CIDADANIA	2	1	0	1	0
PSB	2	2	0	0	1
NOVO	1	0	0	1	1
PSC	1	2	0	3	6
REDE	1	1	0	0	0
SOLIDARIEDADE	1	1	0	1	0
AVANTE	0	0	0	1	0
PATRIOTA	0	1	0	1	2
PDT	0	1	0	0	0
Total Right	37	32	54	44	64
Total CenterRight	14	20	1	26	0
Total Left	19	18	15	0	6

Note: Parties are allocated either as 'left' (red), 'right' (cyan) or 'center-right' (cyan) according to Bolognesi et al. (2022). Column 1 presents the number of representatives elected by party in the actual open-list system. Columns 2 to 5 show the 70 candidates elected in the most number of the 1,102 simulated constituency maps: column 2 ranks candidates by open-list vote; column 3 aggregates by party in each simulated district; column 4 aggregates by left/right; and column 5 aggregates into three ideological blocks.