First-price sealed-bid auction with secret reservation price

Article *in* Annales d Économie et de Statistique · April 1994

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ANNALES D'ÉCONOMIE ET DE STATISTIQUE. – N° 34 — 1994

First-Price Sealed-Bid Auctions With Secret Reservation Prices

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ABSTRACT. – In this paper we consider a first-price sealed bid auction with a secret reservation price. Such auctions are used frequently in France to sell timber. Within the independent private values paradigm, we show that the equilibrium strategy of the seller is to choose a reservation price equal to his private value. We characterize the symmetric Bayesian equilibrium strategy for the buyers as the solution of a differential equation. We also show that a strategy of public reservation price is better for the seller than a strategy of secret reservation price.

To evaluate the expected gain for the seller from moving from a secret to the optimal public reservation price, we estimate the model using data from an actual auction of timber. First, we solve the problem of identification of the underlying distributions of private values. Then we propose a two-step structural nonparametric estimation method to shed some lights on the shapes of these distributions. This is used to formulate and estimate a parsimonious and structural parametric model that reproduces the salient features of our nonparametric analysis.

Enchères cachetées au premier prix avec prix de retrait secret

RÉSUMÉ. – Nous considérons une enchère au premier prix sous pli fermé avec prix de réserve secret. De telles enchères sont souvent utilisées en France pour vendre le bois. Dans le paradigme des valeurs privées indépendantes, nous montrons que la stratégie d'équilibre du vendeur est de choisir un prix de réserve égal a sa valeur privée. Nous caractérisons la stratégie d'équilibre Bayésien symétrique des acheteurs comme la solution d'une équation différentielle. Nous montrons aussi que la stratégie de prix de réserve public du vendeur est meilleure que la stratégie de prix de réserve secret.

Pour évaluer le gain espéré pour le vendeur du passage d'un prix de réserve secret à un prix public, nous estimons le modèle structurel fourni par les stratégies d'équilibre. D'abord, nous résolvons le problème d'identification des distributions des valeurs privées. Ensuite, nous proposons une méthode d'estimation non paramétrique structurelle à deux étapes pour obtenir une idée de la forme des distributions. Cela est ensuite utilisé pour formuler et estimer un modèle structurel paramétrique.

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1 Introduction

Auctions provide a wealth of data, hardly explored until today, that can be exploited in a structural econometric approach based on the new developments of industrial organization. See, however, the pioneering empirical work by HANSEN [1985, 1986], HENDRICKS, Porter and BOUDREAU [1989], HENDRICKS and PORTER [1988] and PAARSCH [1991, 1992]. These theoretical developments emphasize asymmetric information and strategic behavior, which are two essential ingredients of auctions.

Recently, LAFFONT, OSSARD and VUONG [1992] have proposed a simulated nonlinear least squares method to estimate the underlying distribution of willingnesses to pay, which explains the bidding behavior in a first-price auction. In such an auction, under the assumption of independent valuations, the symmetric Bayesian equilibrium strategies of the game induced are solvable in an explicit form. Unfortunately, most variations on these assumptions lead to situations where equilibrium strategies are defined only in an implicit form. This is the case examined in LAFFONT and VUONG [1994] where the asymmetry between buyers is raising this difficulty.

Similarly, Bayesian equilibrium strategies are not solvable explicitly in a class of auctions that are widely used in France to sell timber. These are first-price auctions with *secret* reservation prices. Since the seller tenders a sealed bid, which is opened at the same time as the buyers' sealed bids, the seller is not much different from a buyer except that his objective function and eventually the distribution of his valuation are different. This simple variation is sufficient to raise the head of asymmetric auctions.

In Section 2 we derive the optimal strategies of the buyer and of the seller for such a first-price auction with secret reservation price within the independent values paradigm. For risk neutral agents, we prove that a policy of publicly announced reservation price would be better for the seller and we obtain the expected gain of moving from the optimal secret reservation price rule to the optimal publicly announced reservation price rule.

Section 3 describes the data coming from 1990 auctions of timber held in the south of France. A structural nonparametric method is developed in Section 4 to uncover the shapes of the distributions of valuations for the sellers and the buyers. These distributions are the basic structural ingredients of our theoretical auction model. This allows us to select some reasonable and parsimonious parametric representations of these distributions conditional upon the observable exogenous variable (the percentage of saw timber). Nonlinear least squares estimation of the parametric representation is then performed in Section 5. The method involves at each step the numerical integration of the differential equation defining the buyers' strategy. These results are used to evaluate the expected gain for sellers from moving to a policy of a publicly announced optimal reservation price.

2 The Model

We consider the first-price auction of an indivisible commodity with I buyers and one seller. The seller has a valuation v_0 for the commodity and submits a sealed reservation price to the organizer of the auction. Buyer i has a willingness to pay for the commodity of v_i, $i = 1, \ldots, \mathrm{I}$, and submits a sealed bid. All agents are risk neutral.

We assume that buyers' valuations $v_1, \ldots, v_\mathrm{I}$ are independently drawn from the same distribution on $[\underline{v}, \overline{v}]$ with cumulative distribution function (cdf) $\mathrm{F}(\cdot)$ and density function $f(\cdot)$ while the seller's valuation v_0 is independently drawn from a distribution on the same support $[\underline{v}, \overline{v}]$ but with a cdf $\mathrm{G}(\cdot)$ and density function $g(\cdot)$. This stochastic structure is common knowledge.

Let $p(\cdot)$ be the seller's strategy, which specifies for each valuation v_0 a secret reservation price $p(v_0)$, and let $s_i(\cdot)$ be buyer i's strategy, which specifies for each willingness to pay v_i a sealed bid $s_i(v_i)$. We want to describe the outcomes of the auction by a Bayesian Nash equilibrium of the game induced by the auction. By symmetry, we restrict the analysis to equilibria in which all buyers have the same strategy, i.e. $s_i(\cdot) = s(\cdot)$ for any i, and we look for equilibria with strictly increasing differentiable strategies $p(\cdot)$ and $s(\cdot)$.

It is intuitively clear that the seller's strategy reduces to $p(v_0) = v_0$ since any higher reservation price would not be credible as it would give up valuable exchanges without any compensatory gain. Formally, if $y \equiv \max_{i=1,\ldots,\mathrm{I}} v_i$ and $\mathrm{F}_{\max}(y) = \mathrm{F}^\mathrm{I}(y)$ denotes its cdf, then using the strict monotony of $s(\cdot)$ the expected utility of the seller is

$$\begin{aligned} V(v_0, \tilde{v}_0) &= v_0 \, \mathrm{P}r\,(s(y) < p(\tilde{v}_0)) + \int_{\{y: s(y) \geqq p(\tilde{v}_0)\}} s(y)\, d\mathrm{F}_{\max}(y) \\ &= v_0 \, \mathrm{F}_{\max}[s^{-1}(p(\tilde{v}_0))] + \int_{s^{-1}(p(\tilde{v}_0))}^{\overline{v}} s(y)\, d\mathrm{F}_{\max}(y) \end{aligned}$$

when the seller deviates with $p(\tilde{v}_0)$ from his strategy $p(v_0)$. The strategy $p(\cdot)$ is optimal if no deviation is profitable. Hence, the necessary first-order condition is

$$\begin{aligned} 0 =& v_0 \, f_{\max}[s^{-1}(p(\tilde{v}_0))]\, s^{-1\prime}(p(\tilde{v}_0)) \frac{dp(\tilde{v}_0)}{dv_0} \\ &- s[s^{-1}(p(\tilde{v}_0))]\, f_{\max}[s^{-1}(p(\tilde{v}_0))]\, s^{-1\prime}(p(\tilde{v}_0)) \frac{dp(\tilde{v}_0)}{dv_0} \end{aligned}$$

at $\tilde{v}_0 = v_0$. This gives $v_0 = s[s^{-1}(p(v_0))] = p(v_0)$.

As the local second-order condition is satisfied and the utility function fulfills the Spence-Mirrlees condition (see Guesnerie and Laffont (1984)), the strategy $p(v_0) = v_0$ is indeed optimal if we find that the optimal strategy of the buyers is strictly increasing as postulated.

Given the seller's strategy, buyer i's expected utility is

$$\mathrm{U}(v_i, \tilde{v}_i) = [v_i - s(\tilde{v}_i)] \, \mathrm{P}\,r \, [s(\tilde{v}_i) > s(v_j), \; j \neq i \text{ and } s(\tilde{v}_i) > v_0]$$

when buyer i deviates by $s(\tilde{v}_i)$ from his strategy $s(v_i)$. Using the strict monotony of $s(\cdot)$ we have

$$\mathrm{U}(v_i, \tilde{v}_i) = [v_i - s(\tilde{v}_i)] \, \mathrm{F}^{\mathrm{I}-1}(\tilde{v}_i) \, \mathrm{G}(s(\tilde{v}_i)).$$

The first-order condition for the optimality of $s(\cdot)$ is:

$$(1) \quad \frac{ds(v_i)}{dv_i} \mathrm{F}(v_i) \, \mathrm{G}(s(v_i)) = [v_i - s(v_i)] \left[(\mathrm{I}-1) \, \mathrm{G}(s(v_i)) \, f(v_i) + \mathrm{F}(v_i) \, g(s(v_i)) \frac{ds(v_i)}{dv_i} \right]$$

Again, the second-order conditions are satisfied if we have a strictly increasing strategy $s(\cdot)$.

To solve the differential equation (1) we need a boundary condition. If buyer i does not bid he has a utility level of zero. If there is a value $\underline{\underline{v}}$ below which he does not bid it must be the case that $s(\underline{\underline{v}}) = \underline{\underline{v}}$ to ensure the continuity of the utility level (required from incentive compatibility). But $\underline{\underline{v}} > \underline{v}$ cannot be an optimal strategy since for $v \in [\underline{v}, \underline{\underline{v}}]$ the buyer's expected utility is strictly positive if the bids are below v and above $\underline{v}$. Hence $\underline{\underline{v}} = \underline{v}$ and $s(\underline{v}) = \underline{v}$ since any lower bid cannot win and any higher bid loses utility.

To summarize, we are interested in the solution of (1) with the boundary condition $s(\underline{v}) = \underline{v}$ where $s(\cdot)$ is an increasing function.

Note, however, that contrary to the classical case of a first price auction with announced reservation price, the optimal strategy cannot be obtained explicitly in general. As a result, the differential equation (1) will have to be integrated numerically. In fact, as it is done in Section 5, it will be more convenient to define the solution $s(\cdot)$ of (1) with boundary condition $s(\underline{v}) = \underline{v}$ implicitly as

$$(2) \qquad s(v) = v - \frac{\displaystyle\int_{\underline{v}}^{v} \mathrm{F}^{\mathrm{I}-1}(u) \, \mathrm{G}(s(u)) \, du}{\mathrm{F}^{\mathrm{I}-1}(v) \, \mathrm{G}(s(v))}.$$

Equation (2) is obtained by multiplying both sides of (1) by $\mathrm{F}^{\mathrm{I}-2}(v)$ and by noting that (1) can be rewritten as

$$\frac{d}{dv}[s(v) \, \mathrm{F}^{\mathrm{I}-1}(v) \, \mathrm{G}(s(v))] = v \frac{d}{dv}[\mathrm{F}^{\mathrm{I}-1}(v) \, \mathrm{G}(s(v))].$$

Then (2) follows by integrating the latter equation from $\underline{v}$ to v and by using the boundary condition $s(\underline{v}) = \underline{v}$.

Clearly, (2) imposes some conditions on $\mathrm{F}(\cdot)$ and $\mathrm{G}(\cdot)$ for $s(\cdot)$ to be strictly increasing as required. Hereafter, we assume that such conditions are satisfied and thus, we restrict our analysis to distributions of valuations such that $s(\cdot)$ defined by (2) is increasing [1]. Also, though similar, (2) differs from the equilibrium strategy in a first-price sealed-bid auction with $\mathrm{I}+1$ bidders and no reservation price in two aspects. First, one player (here the seller) draws his private value from a distribution $\mathrm{G}(\cdot)$ that may differ from the distribution $\mathrm{F}(\cdot)$ of private values of the I other bidders. Second and more importantly, the equilibrium strategy $s(\cdot)$ enters the right-hand side and hence (2) defines $s(\cdot)$ only implicitly.

Next, an important question is whether a seller benefits by keeping the reservation price secret rather than making it public. In view of the next proposition one may wonder why a secret reservation price is being used [2].

PROPOSITION 1: A choice of public reservation price is always as good and in general better for the seller than a choice of private reservation price.

Proof: Let v be the winner's valuation. Consider the ex ante expected utility of the seller when keeping the reservation price secret. Specifically, taking first the conditional expectation given $\max_i v_i = v$, we have

$$\text{(3)} \qquad \begin{aligned} &\mathrm{E}_{v_0}\left[v_0 \mathbf{1}(s(v) < v_0) + s(v)\mathbf{1}(s(v) > v_0)\right] \\ &= \int_{s(v)}^{\overline{v}} v_0 \, d\mathrm{G}(v_0) + s(v)\,\mathrm{G}(s(v)), \end{aligned}$$

where $\mathrm{E}_{v_0}[\cdot]$ denotes the expectation with respect to v_0, $s(\cdot)$ is the solution described by (2) and where we have used the result obtained above that the optimal reservation price rule is $p(v_0) = v_0$. Then, on average, the utility of the seller when keeping the reservation price secret is the integral of (3) over v with respect to the distribution of $\max_i v_i$.

Consider now the case of a public reservation price. We will exhibit a public reservation price strategy that yields the same expected profit as the above secret reservation rule. Let

$$\text{(4)} \qquad p(v_0) = \begin{cases} s^{-1}(v_0) & \text{if} \quad v_0 \leq s(\overline{v}), \\ \overline{v} & \text{if} \quad v_0 > s(\overline{v}). \end{cases}$$

Let $e(\cdot, p(v_0))$ denote the buyers' Bayesian Nash equilibrium strategy in a first-price auction with public reservation price $p(v_0)$. We have

$$\text{(5)} \quad e(v, p(v_0)) = v - \frac{1}{\mathrm{F}^{\mathrm{I}-1}(v)} \int_{p(v_0)}^{\overline{v}} \mathrm{F}^{\mathrm{I}-1}(u)\, du \qquad \text{for} \quad v \geq p(v_0)$$

1. Proposition 2 in Section 4.1 gives a necessary and sufficient condition on the distribution of observed bids and reservation prices for the existence of such a strictly increasing equilibrium strategy.
2. We are aware of recent explanations based on risk aversion given by Tan [1991] and Vincent [1993], but these explanations are not of great appeal in our particular case, where buyers are firms and the auctioned items are small relative to the firms' activities. Furthermore, Vincent's example concerns a common value auction.

(see, e.g., RILEY-SAMUELSON [1981]). Distinguishing the cases where $v_0 > s(\overline{v})$ and $v_0 < s(\overline{v})$, then (4) and (5) give the expected utility of the seller given $\max_i v_i = v$ as

$$
\begin{aligned}
(6)\quad & \mathrm{E}_{v_0}\left[v_0\,\mathbf{1}\,(v < p(v_0)) + e(v, p(v_0))\,\mathbf{1}\,(v \geq p(v_0))\right] \\
&= \int_{s(\overline{v})}^{\overline{v}} v_0\,d\mathrm{G}(v_0) + \int_{\underline{v}}^{s(\overline{v})} v_0\,\mathbf{1}\,(v < s^{-1}(v_0))\,d\mathrm{G}(v_0) \\
&\quad + \int_{\underline{v}}^{s(\overline{v})} \left(v - \frac{1}{\mathrm{F}^{\mathrm{I}-1}(v)} \int_{s^{-1}(v_0)}^{v} \mathrm{F}^{\mathrm{I}-1}(u)\,du\right) \\
&\quad \times \mathbf{1}\,(v \geq s^{-1}(v_0))\,d\mathrm{G}(v_0) \\
&= \int_{s(\overline{v})}^{\overline{v}} v_0\,d\mathrm{G}(v_0) + \int_{s(v)}^{s(\overline{v})} v_0\,d\mathrm{G}(v_0) \\
&\quad + \int_{\underline{v}}^{s(\overline{v})} \left(v - \frac{1}{\mathrm{F}^{\mathrm{I}-1}(v)} \int_{s^{-1}(v_0)}^{v} \mathrm{F}^{\mathrm{I}-1}(u)\,du\right) d\mathrm{G}(v_0) \\
&= \int_{s(v)}^{\overline{v}} v_0\,d\mathrm{G}(v_0) + v\,\mathrm{G}(s(v)) \\
&\quad - \frac{1}{\mathrm{F}^{\mathrm{I}-1}(v)} \int_{\underline{v}}^{s(v)} \mathrm{F}^{\mathrm{I}-1}(s^{-1}(v_0))\,\mathrm{G}(v_0)\,\frac{1}{s'(s^{-1}(v_0))}\,dv_0 \\
&= \int_{s(v)}^{\overline{v}} v_0\,d\mathrm{G}(v_0) + v\,\mathrm{G}(s(v)) \\
&\quad - \frac{1}{\mathrm{F}^{\mathrm{I}-1}(v)} \int_{\underline{v}}^{v} \mathrm{F}^{\mathrm{I}-1}(u)\,\mathrm{G}(s(u))\,du,
\end{aligned}
$$

where the third equation follows from an integration by parts and the last equation follows from the change of variable $u = s^{-1}(v_0)$ and the boundary condition $s^{-1}(\underline{v}) = \underline{v}$. Then, (2) establishes that (6) is identical to (3).

To complete the proof, it suffices to find examples where a public reservation price rule, such as the optimal one, can be *strictly* better than the secret reservation price rule. Such an example is provided in Section 5. □

The intuition (and also an alternative proof) of this proposition can be given by appealing to the theory of mechanism design. An auction is characterized for a given individual by the expected probability $\pi(v)$ (say) that he will receive the object sold as a function of his announced willingness to pay v. In other words $\pi(\cdot)$ is the action implemented by the mechanism. From incentive theory we know that there exists a unique minimal expected transfer $t(\cdot)$ which implements this "action". The agent's expected utility is then $v\,\pi(\tilde{v}) - t(\tilde{v})$, where $\tilde{v}$ is his announced willingness to pay. The transfert function $t(\cdot)$ is defined by the first-order condition of incentive compatibility

$$\frac{dt}{dv} = v\,\frac{d\pi}{dv}$$

with the boundary condition $t(p(v_0)) = p(v_0)\,\pi(p(v_0))$.

By choosing the public reservation price rule defined in (4) we ensure that the object is sold exactly under the same circumstances as with the secret reservation price rule. Since in both auctions the object is given to the same buyer in the same circumstances, the action implemented for a given buyer is exactly the same in both auctions, i.e. a probability

$$\pi(v) = \begin{cases} F^{I-1}(v) & \text{if} \quad v > p(v_0), \\ 0 & \text{if} \quad v < p(v_0). \end{cases}$$

Consequently the expected transfer which can be obtained from a given buyer in both auctions is the same.

The proof of Proposition 1 uses a public reservation price rule (4) that is not necessarily optimal for the seller. Indeed, recall that the optimal public reservation price rule $p^*(v_0)$ solves

$$p = v_0 + \frac{1 - F(p)}{f(p)} \tag{7}$$

for every v_0 (see Laffont and Maskin [1980] and Riley and Samuelson [1981]). In particular, the optimal reservation price rule is independent of the number of buyers, contrary to the rule defined by (4). The comparison between those two rules is not computationally easy because transactions do not occur in the same circumstances, but (7) is by definition at least as good as (4) and in general it will be better in terms of expected revenue for the seller.

After estimating the distributions $G(\cdot)$ and $F(\cdot)$ we will be able to compute the expected gain for the seller of moving from a secret to a public reservation price rule.

3 The Data

In France, standing timbers are sold to private firms (sawmills) through various mechanisms. One of the most frequently used mechanisms is the first-price sealed-bid auction where the seller's reservation price is unknown in advance to the bidders. It is the mechanism that we analyze empirically hereafter.

The particular auction market that we consider is organized by a cooperative, Forestarn, located in Tarn, a southwest region in France. Standing timbers for sale in this market are essentially coniferous and stand on different lots, which are privately own. The cooperative has been set up by the tree owners and the market is held twice a year. The characteristics of each auctioned lot are fully described in a booklet which is freely available to every participant at the auction. Each potential bidder can also inspect the auctioned lots in advance. Each lot is auctioned successively the day of the auction. The first lot is randomly drawn from the available lots.

As in a first-price sealed bid auction, interested sawmill owners tender their sealed bid to the auction organizer. The winner of the auction is the highest bidder provided his bid is higher than the seller's reservation price. The winner pays his bid. Unlike in a standard first-price sealed-bid auction, however, the seller's reservation price is not announced in advance to the bidders though it must be given to the organizer prior to the opening of the market.

The data we use concern the market that occured in 1990. Seventy lots were auctioned. We subscript lots by $l = 1, \ldots, \mathrm{L} = 70$. For each lot we have observations on the seller's reservation price p_{0l}, the number I_l of actual bidders with their bids b_{il}, $i = 1, \ldots, \mathrm{I}_l$, as well as the complete vector x_l of characteristics of the lot as described in the booklet given to the participants. The number of actual bids varies across lots from one to nine bids and there is a grand total of $n = 281$ bids, where $n = \sum_{i=1}^{\mathrm{L}} \mathrm{I}_l$.

Table 1 gives some summary statistics. Reservation prices and bids are measured in French Francs by cubic meter of standing timber. As characteristics, we only report the estimated percentage of saw timber of the lot. This variable is known to be the most important determinant of the value of the lot. Figure 1 provides a plot of the reservation price and the highest bid for each lot.

TABLE 1

Summary Statistics.

Variables	Mean	Std Error	Minimum	Maximum	Obs
Reservation Price	161.26	72.14	42.50	343.30	70
Bids	168.50	70.60	18.60	354.90	281
Percent of Timber	0.54	0.28	0.00	1.00	70

4 Nonparametric Estimation

In this section we propose a structural nonparametric method for estimating the theoretical model of Section 2. The statistical properties of this estimation method such as consistency, asymptotic normality and rate of convergence can be obtained from results in GUERRE, PERRIGNE, SIMIONI and VUONG [1993]. Here, however, we will not attempt to make statistical inferences using this nonparametric method in view of the relative small size of our sample. Instead, we will use our nonparametric estimates to shed some lights on the shapes of the distributions of buyers and sellers private values. This preliminary analysis will be used in Section 5 to formulate and estimate a parsimonious parametric model.

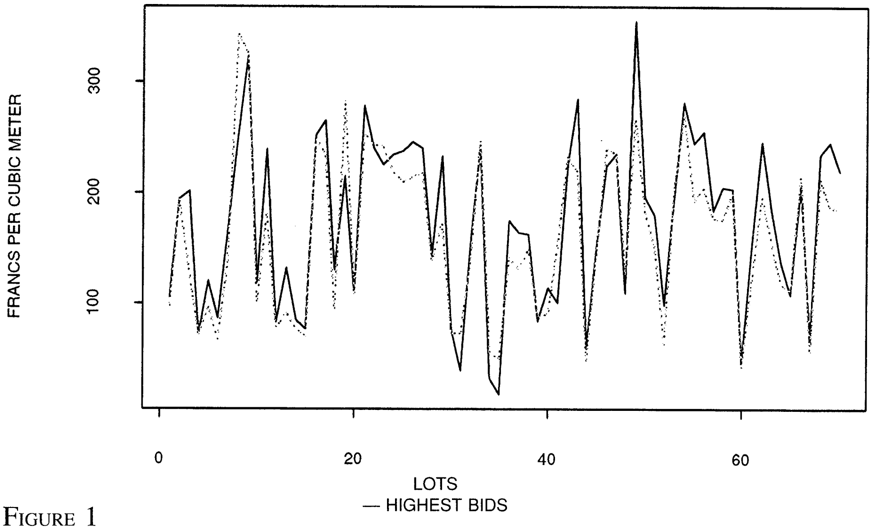

FIGURE 1

Reservation Price and Highest Bid.

4.1. Identification

Structural estimation of the first-price sealed-bid auction model with secret reservation prices consists in estimating the basic ingredients of the theoretical model of Section 2 given the available observations.

The basic ingredients (or structural elements) of the theoretical auction model are the distribution of the seller's private value and the common distribution of bidders' private values. Because auctioned lots are not identical, we will allow these distributions to depend on characteristics x that are known to all participants. Thus, we use hereafter the notation $\mathrm{G}_l \equiv \mathrm{G}(\cdot\,|x_l)$ and $\mathrm{F}_l \equiv \mathrm{F}(\cdot\,|x_l)$ for $l = 1, \ldots, \mathrm{L}$.

The basic idea underlying structural estimation is as follows. Because bids are related to private values, which are random and distributed as F_l for the l-th lot, then the observed bids are also random with some uniquely determined distribution H_l and density h_l on $[\underline{b}_l, \overline{b}_l] = [\underline{v}, s_l(\overline{v})]$. This support follows from the boundary condition $s_l(\underline{v}) = \underline{v}$ and the fact that $s_l(\cdot)$ is strictly increasing in v [3]. The distribution H_l of bidders' private values and its density h_l are

$$(8) \quad \mathrm{H}_l(b) = \mathrm{F}_l(s_l^{-1}(b)) \qquad \text{and} \qquad h_l(b) = \frac{1}{s_l'(s_l^{-1}(b))} f_l(s_l^{-1}(b))$$

for all $b \in [\underline{v}, \overline{b}_l]$.

3. To simplify, we assume that the lower and upper bounds $\underline{v}$ and $\overline{v}$ are independent of l. Note, however, that the upperbound $\overline{b}_l$ typically depends on l even when $\overline{v}$ is independent of l. This raises some technical difficulties with (parametric) maximum likelihood estimation (see DONALD and PAARSCH [1992, 1993]).

Similarly, the reservation price p_{0l} for the l-th lot is related to the seller's private value, which is random and distributed as G_l. Hence the distribution of p_{0l} is uniquely determined. Because the equilibrium strategy of the seller is to set the reservation price p_{0l} equal to his private value v_{0l}, then the distribution of p_{0l} is simply G_l.

We can now turn to the problem of identification, which is one of the most important problems in structural estimation. In the present case this problem reduces to whether the structural elements F_l and G_l are uniquely determined from the knowledge (or estimation) of the distribution of the observed reservation price p_{0l} and the distribution H_l of the observed bids for the l-th lot. For G_l, the problem is trivial since G_l is exactly the distribution of the observed reservation price for the l-th lot.

Identification of the distribution F_l of the buyers' private values for the l-th lot is not as straightforward for at least two reasons. First, the distribution H_l of observed bids $b = s_l(v)$ depends on F_l in two ways: directly through v, which is distributed as F_l, and indirectly through the equilibrium strategy $s_l(\cdot)$, which depends on F_l. Specifically, from the differential equation (1) or the implicit equation (2), the bidders' equilibrium strategy is of the form

$$(9) \qquad s_l(v) = s(v; I_l, F_l, G_l) \qquad \text{for} \quad \underline{v} \leq v \leq \overline{v}$$

with $s_l(\underline{v}) = \underline{v}$, where we have explicitly indicated the dependence of $s_l(\cdot)$ on I_l, F_l and G_l. It is because F_l also enters into the equation relating the observed variable b to the unobserved variable v that Roehrig's [1988] general nonparametric identification results do not apply to the present case and to other auction models, more generally.

Second, an additional difficulty here is that the Bayesian Nash equilibrium strategy $s_l(\cdot)$ cannot be solved explicitly from the first-order differential equation (1). In fact, such a difficulty typically arises as soon as one departs even slightly from the standard auction models (see, e.g., Laffont and Vuong [1994] for another example).

Recalling that we restrict ourselves to strictly increasing differentiable equilibrium strategies, the next proposition gives a necessary and sufficient condition on the distributions of observed bids and reservation prices for the existence of a pair of latent distributions (F_l, G_l). It also solves the identification problem by stating that such a pair is unique whenever it exists. For each l, define the function

$$(10) \qquad \xi_l(b) \equiv \xi(b; I_l, H_l, G_l) = b + \frac{1}{(I_l - 1)\lambda_{hl}(b) + \lambda_{gl}(b)}$$

for $b \in [\underline{v}, \overline{b}_l]$, where $\lambda_{hl}(\cdot) \equiv h_l(\cdot)/H_l(\cdot)$ and $\lambda_{gl}(\cdot) \equiv g_l(\cdot)/G_l(\cdot)$ [4].

4. At the boundaries, $\xi_l(b)$ is defined by $\xi_l(\underline{v}) = \lim_{b \to \underline{v}} \xi_l(b)$ and $\xi_l(\overline{b}_l) = \lim_{b \to \overline{b}_l} \xi_l(b)$. Note that $\xi_l(\underline{v}) = \underline{v}$ because $\lambda_{hl}(\underline{v}) = \lambda_{gl}(\underline{v}) = \infty$.

PROPOSITION 2: Let H_l and G_l be two absolutely continuous distributions with respective supports $[\underline{v}, \bar{b}_l]$ and $[\underline{v}, \bar{v}]$, where $\bar{b}_l < \bar{v} = \xi_l(\bar{b}_l)$. Then there exists a pair of distributions of buyers and seller's private values with common support such that H_l and G_l are the corresponding distributions of equilibrium bids and reservation price in a first-price sealed-bid auction with secret reservation price if and only if the following condition holds:

C: the function $\xi_l(\cdot)$ defined in (10) is strictly increasing and differentiable in b on $[\underline{v}, \bar{b}_l]$.

If this is the case, then the distributions of buyers' and seller's private values are unique and respectively equal to $H_l(\xi_l^{-1}(\cdot))$ and G_l with common support $[\underline{v}, \bar{v}]$. Moreover, $\xi_l(\cdot)$ is the inverse of the equilibrium bid strategy $s_l(\cdot)$.

Proof: We essentially adapt the proof of Proposition 1 in GUERRE, PERRIGNE, SIMIONI and VUONG [1993] to the present case.

To prove that condition C is necessary, recall from Section 2 that the seller's equilibrium strategy is to set the reservation price equal to his private value. Thus, the distribution of the seller's private value must be equal to G_l and hence, exists and is unique. Let F_l denote the distribution of buyers private values. Since G_l has support $[\underline{v}, \bar{v}]$ and F_l and G_l have common support by assumption, then the support of F_l must be $[\underline{v}, \bar{v}]$.

Then, from Section 2, we know that the buyers' equilibrium bid strategy must solve the first-order differential equation (1) with boundary condition $s_l(\underline{v}) = \underline{v}$. For the l-th lot, this differential equation can be rewritten as

$$1 = (v - s_l(v))\left((I_l - 1)\frac{f_l(v)}{F_l(v)}\,\frac{1}{s_l'(v)} + \frac{g_l(s_l(v))}{G_l(s_l(v))}\right) \tag{11}$$

for all $v \in [\underline{v}, \bar{v}]$. On the other hand, from (8) we obtain

$$s_l'(v) = f_l(v)/h_l(b)$$

for all $b = s_l(v)$. Using this equation and (8) in the differential equation (11) and solving for v, we obtain

$$v = b + \frac{1}{(I_l - 1)\,\lambda_{hl}(b) + \lambda_{gl}(b)} = \xi_l(b) \tag{12}$$

for all $b = s_l(v)$ using definition (10) of $\xi_l(\cdot)$. Thus the function $\xi_l(\cdot)$ is nothing else than the inverse function $s_l^{-1}(\cdot)$, i.e.,

$$\xi(b; I_l, H_l, G_l) = s^{-1}(b; I_l, F_l, G_l) \quad \text{for all } b \in [\underline{v}, \bar{b}_l]$$

where $s^{-1}(\cdot)$ denotes the inverse of $s(\cdot)$ as defined in (9) with respect to its first argument. Because $s_l(\cdot)$ is strictly increasing and differentiable, then $\xi_l(\cdot)$ is strictly increasing and differentiable. Thus, condition C must hold.

To prove sufficiency, take the distribution of the seller's private value to be G_l and define the distribution of buyers' private values as

$$F_l(v) = H_l(\xi_l^{-1}(v)) \quad \text{for all } v \in [\underline{v}, \bar{v}]. \tag{13}$$

This is possible since $\xi_l(\cdot)$ is strictly increasing on $[\underline{v}, \bar{b}_l]$ by condition C. Moreover, $\xi_l(\underline{v}) = \underline{v}$ (see footnote 3) and $\xi_l(\bar{b}_l) = \bar{v}$ by assumption. Thus $\xi_l^{-1}(\cdot)$ and hence $\mathrm{F}_l(\cdot)$ are well-defined and strictly increasing on $[\underline{v}, \bar{v}]$. It is easy to check that $\mathrm{F}(\underline{v}) = 0$ and $\mathrm{F}(\bar{v}) = 1$. Hence F_l is a well-defined cdf with same support as G_l, as required.

It remains to show that these distributions of seller's and buyers private values lead to the given distributions of reservation price and bids in a first-price auction with secret reservation price. For G_l, this is straightforward. For H_l, it suffices to show that (8) holds. In view of (13), it suffices to show that $\xi_l^{-1}(\cdot)$ is the buyers' equilibrium strategy when the distribution of the seller's and buyers' private values are F_l and G_l as defined in the preceding paragraph. This can be done by checking that $\xi_l^{-1}(\cdot)$ satisfies the differential equation (11) with boundary condition $\xi_l^{-1}(\underline{v}) = \underline{v}$. Alternatively, it suffices to check that $\xi_l(\cdot)$ satisfies the differential equation for the inverse equilibrium strategy, which is from (11)

$$(11) \qquad 1 = (s_l^{-1}(b) - b)\left((\mathrm{I}_l - 1)\frac{f_l(s_l^{-1}(b))}{\mathrm{F}_l(s_l^{-1}(b))}\, s_l^{-1\prime}(b) + \frac{g_l(b)}{\mathrm{G}_l(b)}\right)$$

with boundary condition $s_l^{-1}(\underline{v}) = \underline{v}$. As noted earlier, the boundary condition is satisfied by $\xi_l(\cdot)$. Moreover, (13) implies $\mathrm{F}_l(\xi_l(b)) = \mathrm{H}_l(b)$ and by differentiating $f_l(\xi_l(b))\,\xi_l'(b) = h_l(b)$. Then, using these equalities and (10) shows that $\xi_l(\cdot)$ satisfies the preceding differential equation. This completes the proof of sufficiency.

Lastly, when they exist, G_l and F_l must be unique. For G_l it is obvious, while for F_l it must satisfies (13) and hence is uniquely defined. Then $\xi_l^{-1}(\cdot)$ must be the buyers' equilibrium strategy. $\square$

Proposition 2 is important for many reasons. First, condition C provides a necessary and sufficient condition for the players (buyers and sellers) to behave as predicted by theory in a first-price sealed-bid auction with secret reservation price. Specifically, if the buyers and sellers adopt the symmetric strictly increasing differentiable Bayesian Nash equilibrium strategies characterized in Section 2, then the distributions of observed bids and reservation prices must be such that the function $\xi_l(\cdot)$ defined in (10) is strictly increasing. Thus, such a property can constitute the basis of a formal test of the theory. This is left for future research.

Second, assuming that the players behave as predicted by the theory of Section 2, then Proposition 2 establishes that the latent distributions F_l and G_l of buyers' and seller's private values are identified from the distributions of observed bids and reservation prices in a first-price sealed-bid auction with secret reservation price. This result is a nonparametric identification result in the sense that no parametric specification of F_l and G_l was necessary. As a result, our identification result necessarily applies to parametric formulations.

Third, it is interesting to note that the function $\xi_l(\cdot)$ can be determined explicitly from the knowledge of I_l, H_l and G_l. Since $\xi_l(\cdot)$ is the inverse of the buyers' equilibrium strategy $s_l(\cdot)$, then the latter can be determined simply by plotting $\xi_l(\cdot)$. In other words, provided one knows I_l, H_l and G_l, one does not have to solve (numerically) the differential equation (1) to determine $s_l(\cdot)$. This remark is important because it underlies the estimation method presented next.

4.2. A Structural Nonparametric Procedure

Our goal is to estimate the structural elements of the theoretical model of Section 2. These elements are I_l, H_l and G_l. The number I_l of bidders is directly observed and hence does not have to be estimated [5]. Estimation of the distribution G_l of sellers' private values is straightforward since the sellers' private values v_{0l} are equal to the reservation prices p_{0l}, which are observed. Thus, assuming that we can estimate the distribution and density of reservation prices, then we immediately obtain estimates $\hat{\mathrm{G}}_l$ and $\hat{g}_l$ of the distribution and density of sellers' private values.

On the other hand, estimation of the distribution F_l of buyers' private values is more involved since it is not identical, in general, to the distribution H_l of observed bids. We can, however, rely upon Proposition 2, which "maps back" the distribution H_l of observed bids to the underlying distribution F_l of private values. Specifically, suppose that estimators $\hat{\mathrm{H}}_l$ and $\hat{h}_l$ of H_l and h_l are available. The basic idea underlying our method is to use (12) to construct a *pseudo sample* of private values corresponding to the observed bids b_{il}. The method is as follows:

Step 1: Estimate H_l, h_l, G_l and g_l by $\hat{\mathrm{H}}_l$, $\hat{h}_l$, $\hat{\mathrm{G}}_l$ and $\hat{g}_l$, respectively. Then estimate the function $\xi_l(\cdot)$ by $\hat{\xi}_l(\cdot)$ using (10), where $\lambda_{hl}(\cdot)$ and $\lambda_{gl}(\cdot)$ are replaced respectively by their estimates $\hat{\lambda}_{hl}(\cdot)$ and $\hat{\lambda}_{gl}(\cdot)$.

Step 2: Use (12) to construct the pseudo sample

$$\{(\hat{v}_{il}, x_l);\ i = 1, \ldots, \mathrm{I}_l,\ l = 1, \ldots, \mathrm{L}\},$$

where $\hat{v}_{il} = \hat{\xi}_l(b_{il})$. Then estimate $\mathrm{F}_l(\cdot)$ and its characteristics using this pseudo sample.

Such an estimation method can be viewed as an *indirect method* [6]. Relative to *direct methods* such as those presented in Laffont and Vuong [1993] for descending auctions with public reservation prices, the present method has the computational advantage of avoiding the numerical integration of the differential equation (1), which here does not have an explicit solution. The method gives nonetheless an estimate of the buyers' equilibrium strategy $s_l(\cdot)$ by plotting $\hat{\xi}_l(\cdot)$. Other important computational advantages are of not requiring the computation of the inverse $\hat{\xi}_l^{-1}(\cdot)$ and of being easily amenable to the estimation of characteristics of interest of the distribution of buyers' private values, such as its conditional expectation.

Up to now, we have not said how H_l, h_l, G_l and g_l are actually estimated from observed bids and reservation prices in Step 1. We have not said either

5. Unlike in some auctions with public reservation prices (see e.g., Laffont, Ossard and Vuong [1991]), the number of potential bidders and the number of actual bidders are identical in a first-price sealed-bid auction with secret reservation price.
6. As mentioned in Guerre, Perrigne, Simioni and Vuong [1993] and as noted by a referee, (13) can be exploited to propose another one-step indirect estimation method. For the reasons given there, we prefer the current two-step method.

how F_l and its characteristics are estimated from the pseudo sample of buyers' private values in Step 2. In this section, following GUERRE, PERRIGNE, SIMIONI and VUONG [1993], we propose a computationally simple two-step nonparametric estimation method and we give below explicit estimators of these distributions and other quantities of interest. Our estimators are based on the kernel method, which has become one of the most widely known nonparametric techniques since its introduction by PARZEN [1962] and ROSENBLATT [1956] (see HÄRDLE [1990] for a recent monograph).

Before doing so, note that parametric estimation of H_l, G_l, F_l, and their densities is possible. Parametric estimation, however, faces the following important difficulty. Typically, the upperbound $\bar{b}_l$ of the support of H_l is (i) finite and (ii) depends on G_l and F_l in a complex way, namely $\bar{b}_l = s(\bar{v}; I_l, F_l, G_l)$. Thus, a correct parametric specification of H_l, G_l and F_l requires that these features be taken into account simultaneously. This greatly restricts the choice of parametric specifications and increase the possibility of misspecification. In contrast, nonparametric estimation presents the convenient feature of letting the data decide upon the shapes of H_l, G_l and F_l so that there is no need for a priori information on the latent distributions of buyers' and sellers private values. Moreover, as we will see, nonparametric estimation is considerably faster in computer time than parametric estimation.

We first consider the nonparametric estimation of the conditional density $g_l(\cdot) = g(\cdot|x_l)$ of private values for the sellers. For this, we use the observations (p_{0l}, x_l), $l = 1, \ldots, L$. Since $p_{0l} = v_{0l}$ and x is a scalar (to simplify), the kernel estimator of the conditional density $g(\cdot\,|\,\cdot)$ is

$$\hat{g}(u|x) = \frac{1}{h_p} \frac{\sum_{l=1}^{L} \phi\left(\frac{u - p_{0l}}{h_p}\right) \phi\left(\frac{x - x_l}{h_x}\right)}{\sum_{l=1}^{L} \phi\left(\frac{x - x_l}{h_x}\right)} \tag{14}$$

for all (u, x), where h_p and h_x are some appropriately chosen scalars called bandwidths, and $\phi(\cdot)$ is a Parzen-Rosenblatt kernel, which is taken hereafter to be the univariate standard normal density.

To estimate the conditional distribution $G(\cdot\,|\,\cdot)$ of sellers' private values, we can integrate $\hat{g}(u|x)$. For a technical reason discussed in GUERRE, PERRIGNE, SIMIONI and VUONG [1993], we prefer the "counting" estimator

$$\hat{G}(u|x) = \frac{\sum_{l=1}^{L} \mathbf{1}(p_{0l} < u)\, \phi\left(\frac{x - x_l}{h_x}\right)}{\sum_{l=1}^{L} \phi\left(\frac{x - x_l}{h_x}\right)} \tag{15}$$

for all (u, x). This gives $\hat{\lambda}_{gl}(\cdot) \equiv \hat{\lambda}_g(\cdot\,|x_l)$, where

$$(16) \qquad \hat{\lambda}_g(u|x) \equiv \frac{\hat{g}(u|x)}{\hat{\mathrm{G}}(u|x)} = \frac{1}{h_p} \frac{\sum_{l=1}^{\mathrm{L}} \phi\left(\frac{u-p_{0l}}{h_p}\right) \phi\left(\frac{x-x_l}{h_x}\right)}{\sum_{l=1}^{\mathrm{L}} \mathbf{1}(p_{0l} < u)\, \phi\left(\frac{x-x_l}{h_x}\right)}$$

for all (u, x). The latter estimator is analogous to standard nonparametric estimators of the hazard rate $g/(1-\mathrm{G})$ (see, e.g., WATSON and LEADBETTER [1964a, 1964b], MURTHY [1965], and HASSANI, SARDA and VIEU [1986] SINGPURWALA and WONG [1993] and for recent surveys).

Similarly, we can estimate nonparametrically $h_l(\cdot) = h(\cdot\,|\,\mathrm{I}_l, x_l)$, $\mathrm{H}_l(\cdot) = \mathrm{H}(\cdot\,|\,\mathrm{I}_l, x_l)$ and $\lambda_{hl}(\cdot) = \lambda_h(\cdot\,|\,\mathrm{I}_l, x_l)$. We must, however, take into account two new features: (i) The distribution H_l of observed bids depends on x_l but also on the number I_l of bidders (see Section 2). This adds I_l as another explanatory variable, which is discrete [7]. (ii) The observations used are now $(b_{il}, \mathrm{I}_l, x_l)$, $i = 1, \ldots, \mathrm{I}_l$, $l = 1, \ldots, \mathrm{L}$. In particular, there are typically more than one bid for the same value of x. These considerations lead to

$$(17) \quad \hat{h}(b|\mathrm{I},x) = \frac{1}{h_p} \frac{\sum_{l=1}^{\mathrm{L}} \mathbf{1}(\mathrm{I}_l = \mathrm{I})\, \phi\left(\frac{x-x_l}{h_x}\right) \left(\frac{1}{\mathrm{I}_l} \sum_{i=1}^{\mathrm{I}_l} \phi\left(\frac{b-b_{il}}{h_b}\right)\right)}{\sum_{l=1}^{\mathrm{L}} \mathbf{1}(\mathrm{I}_l = \mathrm{I})\, \phi\left(\frac{x-x_l}{h_x}\right)}$$

$$(18) \qquad \hat{\mathrm{H}}(b|\mathrm{I},x) = \frac{\sum_{l=1}^{\mathrm{L}} \mathbf{1}(\mathrm{I}_l = \mathrm{I})\, \phi\left(\frac{x-x_l}{h_x}\right) \left(\frac{1}{\mathrm{I}_l} \sum_{i=1}^{\mathrm{I}_l} \mathbf{1}(b_{il} < b)\right)}{\sum_{l=1}^{\mathrm{L}} \mathbf{1}(\mathrm{I}_l = \mathrm{I})\, \phi\left(\frac{x-x_l}{h_x}\right)}$$

$$(19) \quad \hat{\lambda}_h(b|\mathrm{I},x) = \frac{1}{h_b} \frac{\sum_{l=1}^{\mathrm{L}} \mathbf{1}(\mathrm{I}_l = \mathrm{I})\, \phi\left(\frac{x-x_l}{h_x}\right) \left(\frac{1}{\mathrm{I}_l} \sum_{i=1}^{\mathrm{I}_l} \phi\left(\frac{b-b_{il}}{h_b}\right)\right)}{\sum_{l=1}^{\mathrm{L}} \mathbf{1}(\mathrm{I}_l = \mathrm{I})\, \phi\left(\frac{x-x_l}{h_x}\right) \left(\frac{1}{\mathrm{I}_l} \sum_{i=1}^{\mathrm{I}_l} \mathbf{1}(b_{il} < b)\right)}$$

for all (b, I, x), and another bandwidth h_b.

Using (10) we can now estimate the function $\xi_l(\cdot)$ by $\hat{\xi}_l(\cdot) \equiv \hat{\xi}(\cdot\,|\mathrm{I}_l, x_l)$, where

$$(20) \qquad \hat{\xi}(b|\mathrm{I},x) = b + \frac{1}{(\mathrm{I}-1)\,\hat{\lambda}_h(b|\mathrm{I},x) + \hat{\lambda}_g(b|x)}$$

7. The discreteness of I_l, however, is not a problem. Kernel estimation has been extended to such situations (see BIERENS [1987]).

for all (b, I, x), where $\hat{\lambda}_h(b|\mathrm{I}, x)$ and $\hat{\lambda}_g(b|x)$ are given by (19) and (16), respectively. From Proposition 2, (20) provides a nonparametric estimate of the inverse function of the bidders' equilibrium strategy for each auctioned lot. Moreover, (20) can be used to construct a sample of bidders' pseudo private values as

$$\hat{v}_{il} \equiv \hat{\xi}(b_{il}|\mathrm{I}_l, x_l) \tag{21}$$

for $i = 1, \ldots, \mathrm{I}_l$ and $l = 1, \ldots, \mathrm{L}$.

Then, in Step 2 a nonparametric estimate of the distribution of bidders' private values is obtained using the pseudo sample $(\hat{v}_{il}, x_l)$, $i = 1, \ldots, \mathrm{I}_l$ and $l = 1, \ldots, \mathrm{L}$. For instance, the kernel estimate of the conditional density $f(\cdot|\cdot)$ of buyers' private values is

$$\hat{f}(v|x) = \frac{1}{h_v} \frac{\sum_{l=1}^{\mathrm{L}} \phi\left(\frac{x - x_l}{h_x}\right)\left(\frac{1}{\mathrm{I}_l} \sum_{i=1}^{\mathrm{I}_l} \phi\left(\frac{v - \hat{v}_{il}}{h_v}\right)\right)}{\sum_{l=1}^{\mathrm{L}} \phi\left(\frac{x - x_l}{h_x}\right)} \tag{22}$$

for all (v, x), where h_v is another appropriate bandwidth. If desired, a nonparametric estimator of $\mathrm{F}(\cdot|\cdot)$ can be obtained by a formula analogous to (15) and (18). This completes the definition of our two-step nonparametric estimation method, whose statistical properties can be derived from results given in GUERRE, PERRIGNE, SIMIONI and VUONG [1993].

Lastly, as indicated earlier, we are frequently interested in the mean of the sellers' or buyers' private values conditional upon the characteristic x. For instance, this is useful when assessing the effect of x on how sellers and buyers evaluate the auctioned object. Then the use of the pseudo sample is particularly convenient. Let $\mathrm{E_G}(u|x)$ and $\mathrm{E_F}(v|x)$ be these conditional means. Then, taking expectation with respect to the density estimates $\hat{g}(u|x)$ and $\hat{f}(u|x)$ given in (14) and (22), we obtain

$$\hat{\mathrm{E}}_{\mathrm{G}}(u|x) = \frac{\sum_{l=1}^{\mathrm{L}} p_{0l}\, \phi\left(\frac{x - x_l}{h_x}\right)}{\sum_{l=1}^{\mathrm{L}} \phi\left(\frac{x - x_l}{h_x}\right)} \tag{23}$$

$$\hat{\mathrm{E}}_{\mathrm{F}}(v|x) = \frac{\sum_{l=1}^{\mathrm{L}} \phi\left(\frac{x - x_l}{h_x}\right)\left(\frac{1}{\mathrm{I}_l} \sum_{i=1}^{\mathrm{I}_l} \hat{v}_{il}\right)}{\sum_{l=1}^{\mathrm{L}} \phi\left(\frac{x - x_l}{h_x}\right)} \tag{24}$$

for all x. These are the well-known NADARAYA [1964] and WATSON [1964] kernel estimators of the regressions $\mathrm{E_G}(u|x)$ and $\mathrm{E_F}(u|x)$ using the sample $\{(p_{0l}, x_l);\, l = 1, \ldots, \mathrm{L}\}$ and the pseudo sample $\{(\hat{v}_{il}, x_l);\, i = 1, \ldots, \mathrm{I}_l,\, l = 1, \ldots, \mathrm{L}\}$ respectively.

4.3. Empirical Results

We have implemented the preceding two-step nonparametric procedure to estimate the densities of private values for the sellers and the buyers from the data described in Section 3 using the percentage of saw timber as the only explanatory variable x. The choice of all bandwith parameters, namely, h_p, h_b, h_x and h_v were chosen according to the rule of thumb, which gives each h as the standard error of the corresponding variable times the number of observations used (L or n) to the power $-1/5$ (see HÄRDLE [1991]).

It is useful to note that our proposed nonparametric procedure is extremely fast computationally. It took about 1 minute of CPU time on a SUN sparc 4/390 network to compute (i) estimates of the conditional densities $\hat{g}(u|x)$ and $\hat{f}(u|x)$ on grids of (100×11) equally spaced values for (u, x) and (v, x), respectively, (ii) estimates of the inverse $\xi(b|\mathrm{I}, x)$ of the equilibrium strategy for (100×11) equally spaced values for (b, x) and $\mathrm{I} = 3$, (iii) estimates of the regressions $\mathrm{E}_{\mathrm{G}}(u|x)$ and $\mathrm{E}_{\mathrm{F}}(v|x)$ for 11 equally space values of x. This is because at no place, we had to solve numerically the differential equation (1) for the bidders' equilibrium strategy. Moreover, there was no need to invert the equilibrium strategy. Lastly, unlike nonlinear parametric estimation methods, nonparametric methods require neither equation solving nor optimization algorithms.

Figures 2 and 3 display the conditional density surfaces for the private values of the sellers and the bidders as functions of these values and the percentage of saw timber. It is interesting to note that both estimated conditional densities are unimodal and are log-normal shaped for every value of x. Moreover, in both cases, the conditional mode appears to increase monotonically with x, which is reasonable.

Figure 4 displays the estimated equilibrium bid function for every value of x and $\mathrm{I} = 3$ (approximately the sample average of the number I_l of bidders across lots). As required, the estimated strategy starts from $\underline{v} = 0$ and is such that $\hat{s}(v;\, \mathrm{I}, \hat{\mathrm{F}}(\cdot\,|x), \hat{\mathrm{G}}(\cdot\,|x)) \leq v$ for all (v, x). Moreover, it is strictly increasing in the private value v. As noted in Section 4.1, this is quite satisfying since strict monotony of the buyers' equilibrium strategy is not imposed in our nonparametric procedure, while it must be satisfied by a strategy that is consistent with the theoretical model of Section 2.

Lastly, Figure 5 displays the nonparametric estimates (23) and (24) of the regressions $\mathrm{E}_{\mathrm{G}}(u|x)$ and $\mathrm{E}_{\mathrm{F}}(v|x)$. The R^2 of these nonparametric regressions are 0.84 and 0.76, respectively, which is quite satisfying. Both are monotonically increasing in the percentage of saw timber. This agrees with the behavior of the conditional mode mentioned earlier. Moreover, both curves are relatively close to each other with buyers slightly overevaluating on average the auctioned lot relative to the sellers.

Figure 5 also displays the nonparametric kernel regression estimate of $\mathrm{E}_{\mathrm{F}}(y|x)$, where $y = \max_{i=1,\ldots,\mathrm{I}_l} v_i$, i.e., the expectation of the highest valuations as a function of timber percentage x. It is obtained by using the pseudo highest private value $\hat{y} = \max_{i=1,\ldots,\mathrm{I}_l} \hat{v}_i$. The R^2 of this nonparametric regression is 94.31. As expected, this estimate is above that of $\mathrm{E_F}\,(v|x)$. It is also substantially above the expectation $\mathrm{E_G}\,(u|x)$ of the sellers.

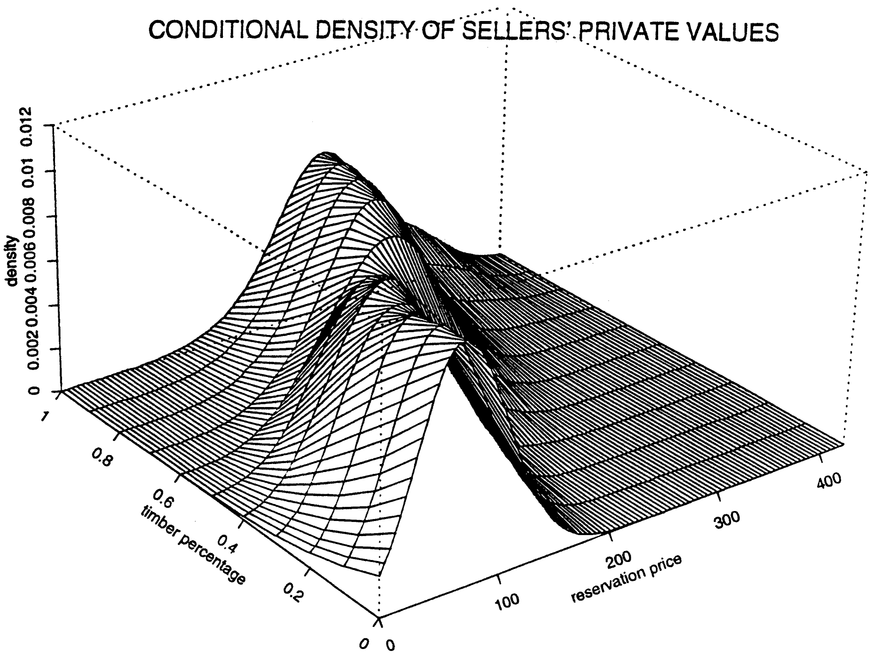

FIGURE 2

Conditional Density of Sellers' Private Values.

5 Parametric Estimation

The purpose of this section is to specify and estimate a parsimonious parametric model that can reproduce the salient empirical features uncovered by our nonparametric analysis of the previous section.

5.1. Parametric Specification

In view of Figures 2 and 3, we choose a log-normal specification for the distributions of private values for the sellers and the buyers. That is, for any given value x of the percentage of saw timber, the specified conditional

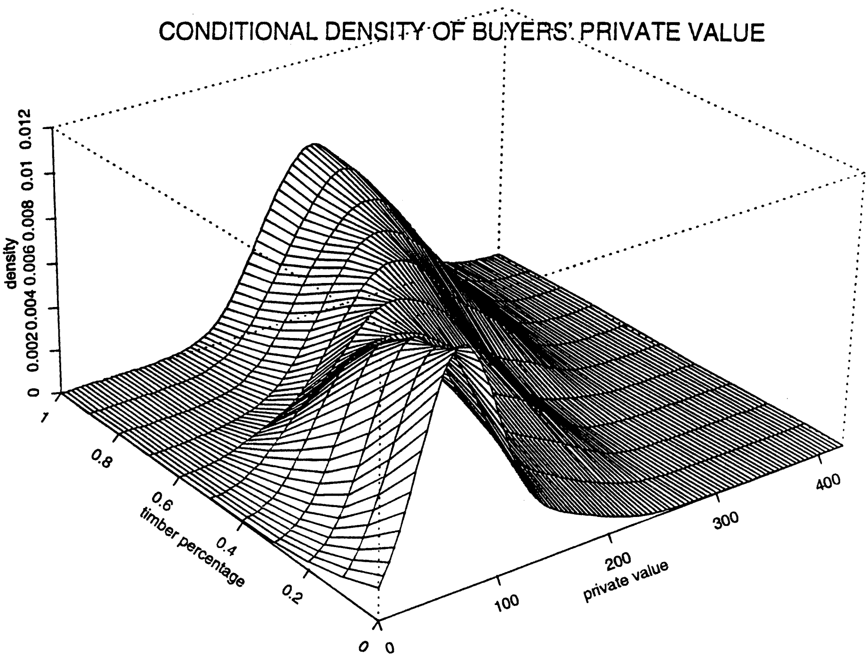

FIGURE 3

Conditional Density of Buyers' Private Value.

densities $g(\cdot|x)$ and $f(\cdot|x)$ are of the form

$$g(u|x) = \frac{1}{u\,\sigma_g(x)\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{\log u - \mu_g(x)}{\sigma_g(x)}\right)^2} \tag{25}$$

$$f(v|x) = \frac{1}{v\,\sigma_f(x)\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{\log v - \mu_f(x)}{\sigma_f(x)}\right)^2} \tag{26}$$

for all $u \geq 0$, $v \geq 0$, $x \geq 0$ and some functions $\mu_g(x)$, $\sigma_g(x)$, $\mu_f(x)$ and $\sigma_f(x)$.

To complete the parametric model, it remains to specify the latter functions. Figures 2 and 3 suggest that the conditional variances of the distributions of sellers' and buyers private values remain about constant. Figure 5 indicates that the conditional means of these private values monotonically increase in the percentage of timber x. To take into account these features and for parsimony reasons, we consider the following simple parametric specifications for the conditional means and variances of sellers and buyers private values

$$\mathrm{E}_g(u|x) = \alpha_g + \beta x \quad \text{and} \quad \mathrm{E}_f(v|x) = \alpha_f + \beta x \tag{27}$$

$$\mathrm{var}_g(u|x) = \gamma_g^2 \quad \text{and} \quad \mathrm{var}_f(v|x) = \gamma_f^2 \tag{28}$$

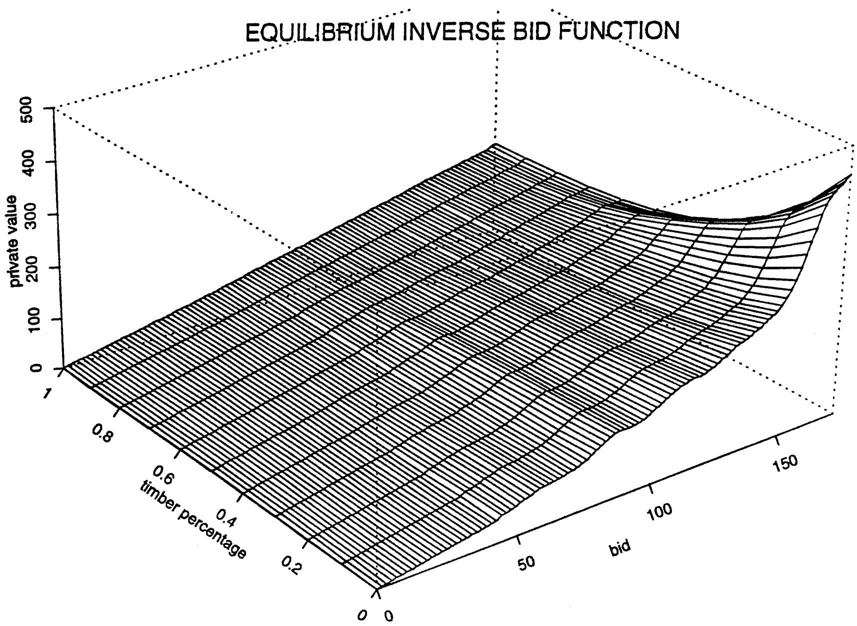

FIGURE 4

Equilibrium Inverse Bid Function.

where $(\alpha_g, \gamma_g, \alpha_f, \gamma_f, \beta)$ are some parameters to be estimated. Note that we have imposed that the slopes in the expectations are identical for sellers and buyers, as suggested by Figure 5.

From the properties of the log-normal distribution, it follows that

$$(29) \qquad \mu_g(x) = \log \frac{\mathrm{E}_g(u|x)}{(1+\tau_g)^{1/2}} \qquad \text{and} \qquad \mu_f(x) = \log \frac{\mathrm{E}_f(v|x)}{(1+\tau_f)^{1/2}}$$

$$(30) \qquad \sigma_g^2(x) = \log(1+\tau_g) \qquad \text{and} \qquad \sigma_f^2(x) = \log(1+\tau_f)$$

where $\tau_g \equiv \mathrm{var}_g(u|x)/\mathrm{E}_g^2(u|x)$ and $\tau_f \equiv \mathrm{var}_f(v|x)/\mathrm{E}_f^2(v|x)$.

To summarize, our parametric model is defined by the conditional densities (25)-(26) for the sellers and buyers private values where the functions $\mu_g(x)$, $\sigma_g(x)$, $\mu_f(x)$ and $\sigma_f(x)$ are defined by (29)-(30), and the parameters are $\theta \equiv (\alpha_g, \gamma_g, \alpha_f, \gamma_f, \beta)$.

5.2. Estimation Method

We estimate our parametric model by Nonlinear Least Squares (NLLS). We need the expectation of the endogenous variables, which are the reservation price and the individual bids. Because the equilibrium reservation

FIGURE 5

Conditional means of Sellers' and Buyers' Private Values.

price p_{0l} is equal to the seller's private value v_{0l}, then its expectation is simply

$$m_{gl} \equiv \mathrm{E_G}\left(u|x_l\right) = \alpha_g + \beta\, x_l. \tag{31}$$

For the individual bids, we have $b = s_l\left(v\right)$, where v is distributed as $f_l\left(\cdot\right) = f\left(\cdot\,|x_l\right)$ as defined in (26) and $s_l\left(\cdot\right)$ is the bidders' equilibrium strategy for the l-th lot. Thus the expectation of individual bids for the l-th lot is

$$m_{fl} = \int_0^{+\infty} s_l\left(v\right) f_l\left(v\right) dv. \tag{32}$$

We estimate simultaneously by NLLS the equations for p_{0l}, $l = 1, \ldots,$ L and b_{il}, $i = 1, \ldots, \mathrm{I}_l$, $l = 1, \ldots,$ L, by minimizing

$$\mathrm{Q}\left(\theta\right) \equiv \sum_{l=1}^{\mathrm{L}} \left(p_l - m_{gl}\right)^2 + \sum_{l=1}^{\mathrm{L}} \sum_{i=1}^{\mathrm{I}_l} \left(b_{il} - m_{fl}\right)^2 \tag{33}$$

with respect to $\theta = \left(\alpha_g, \gamma_g, \alpha_f, \gamma_f, \beta\right)$. Clearly, there are many alternative but computationally more demanding estimation methods, some of which are asymptotically more efficient, such as weighted NLLS. Given the computational difficulties already associated with NLLS, which are discussed next, we only consider this estimator.

The main computational difficulty with the minimization of $Q(\theta)$ is that the equilibrium strategy $s_l(\cdot)$ cannot be obtained in explicit form [8]. In addition, this strategy depends on $g_l(\cdot)$ and $f_l(\cdot)$, which depends themselves on the parameter vector θ. This implies that, for every $l = 1, \ldots, L$ and every trial value for θ, the differential equation (1) must be solved numerically. Then, for each l, the (approximate) solution must be numerically integrated over $[0, +\infty]$ with respect to $f_l(\cdot)$ so as to evaluate m_{fl}.

To minimize somewhat the considerable amount of computations, we have used a numerical algorithm that solves indirectly the differential equation (1). This algorithm relies on (2), which gives implicitly the equilibrium bid strategy. For the l-th lot, this equation becomes

$$s_l(v) = v - \frac{\displaystyle\int_0^v \mathrm{M}_l(a, s_l(a))\, da}{\mathrm{M}_l(v, s_l(v))} \tag{34}$$

since $\underline{v} = 0$, where $\mathrm{M}_l(a, w) \equiv \mathrm{F}_l^{\mathrm{I}_l - 1}(a)\, \mathrm{G}_l(w)$. For any arbitrary chosen value v, this suggests to solve numerically (34) for $s_l(v)$. Then for every trial value for θ and every l, $s_l(\cdot)$ can be integrated with respect to $f_l(\cdot)$.

In fact, since numerical integration is necessary for evaluating m_{fl}, we need to evaluate $s_l(\cdot)$ only at a sufficiently large number K of points $c_1, \ldots, c_{\mathrm{K}}$. Then to minimize CPU time, we can exploit (34) to obtain the recursive relation

$$\begin{aligned} &s_l(c_{k+1}) \\ &= c_{k+1} + (s_l(c_k) - c_k)\, \frac{\mathrm{M}_l(c_k, s(c_k))}{\mathrm{M}_l(c_{k+1}, s_l(c_{k+1}))} - \frac{\displaystyle\int_{c_k}^{c_{k+1}} \mathrm{M}_l(a, s_l(a))\, da}{\mathrm{M}_l(c_{k+1}, s_l(c_{k+1}))} \end{aligned}$$

for $k = 1, \ldots, \mathrm{K} - 1$ with $c_0 \equiv 0$. Then, starting from $s_l(c_0) = 0$, we can determine $s_l(c_{k+1})$ from $s_l(c_k)$ by means of a fixed point algorithm. To simplify further computations, $\displaystyle\int_{c_k}^{c_{k+1}} \mathrm{M}_l(a, s_l(a))\, da$ is evaluated by quadrature.

Even with these numerical simplifications, NLLS estimation of our simple parametric model with 5 parameters, 70 lots and 281 bids took about 100 hours of CPU time on a SUN Sparc 4/390 network. This is to be compared to the 1 minute of CPU time for the preceding nonparametric indirect estimation method of Section 4.

8. This prevents us from using the simulated NLLS estimation method developed in LAFFONT, OSSARD and VUONG [1991] as well as other methods based on simulated moments presented in LAFFONT and VUONG [1993].

5.3. **Empirical Results**

Table 2 gives the NLLS estimates of $\theta \equiv (\alpha_g, \gamma_g, \alpha_f, \gamma_f, \beta)$ with WHITE [1980] heteroskedastic consistent standard errors in parentheses.

TABLE 2

NLLS Estimation.

Variables	Sellers		Buyers
Constant	37.6		122.5
	(2.9)		(12.5)
Percent of Timber		230.8	
		(7.1)	
γ	42.2		320.1
	(1.0)		(65.3)
number of lots: 70			
number of bids: 281			

As expected, the percentage of saw timber is highly significant and its effect is positive on the buyers' and sellers' value of the auctioned lot.

In terms of goodness-of-fit, the R^2 for the sellers' reservation prices is 0.80, while the R^2 for the buyers' bids is 0.76. These R^2 are not significantly smaller than the R^2 of our nonparametric regressions of Section 4.3. This is quite satisfying given the small number of estimated parameters. Figure 6 displays the observed and predicted reservation prices on our L lots. Figure 7 displays the observed and estimated highest bids. These figures show that the observations are relatively well predicted by our parsimonious structural parametric model.

6 Conclusion

This paper has proposed a method for estimating the underlying distributions of willingnesses to pay for both buyers and sellers as functions of the percentage of saw timber in first-price sealed-bid auctions with secret reservation prices. Specifically, we have propose to apply a computationally attractive two-step nonparametric indirect estimation procedure to uncover the shapes of these distributions. Then we have used these findings to formulate a parsimonious parametric model for these distributions, which are thus estimated by nonlinear least squares using numerical integration of the differential equation characterizing the bidders' equilibrium strategies.

Because our estimation approach is fully structural, our estimation results of Section 5 can be used to evaluate, for instance, the gain sellers would

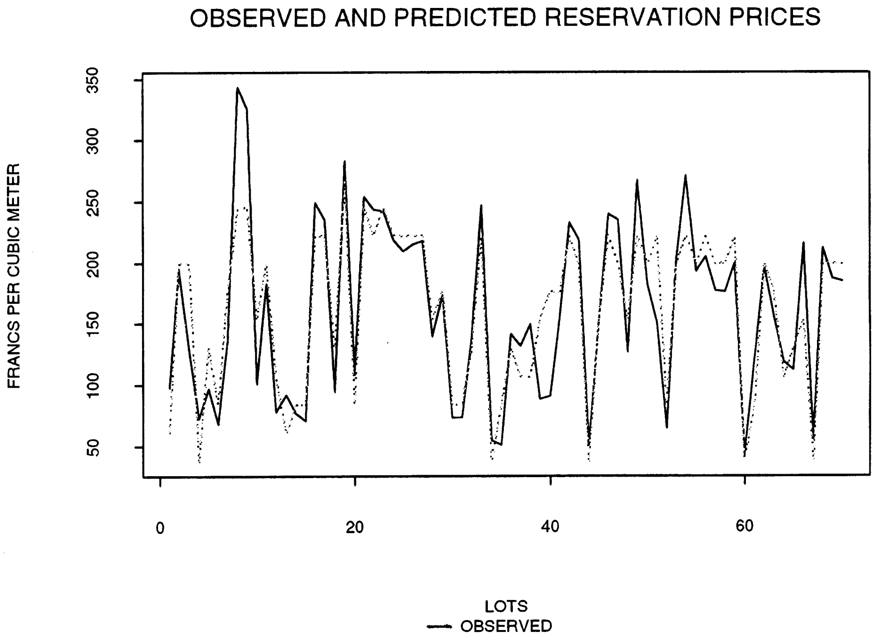

FIGURE 6

Observed and Predicted Reservation Prices.

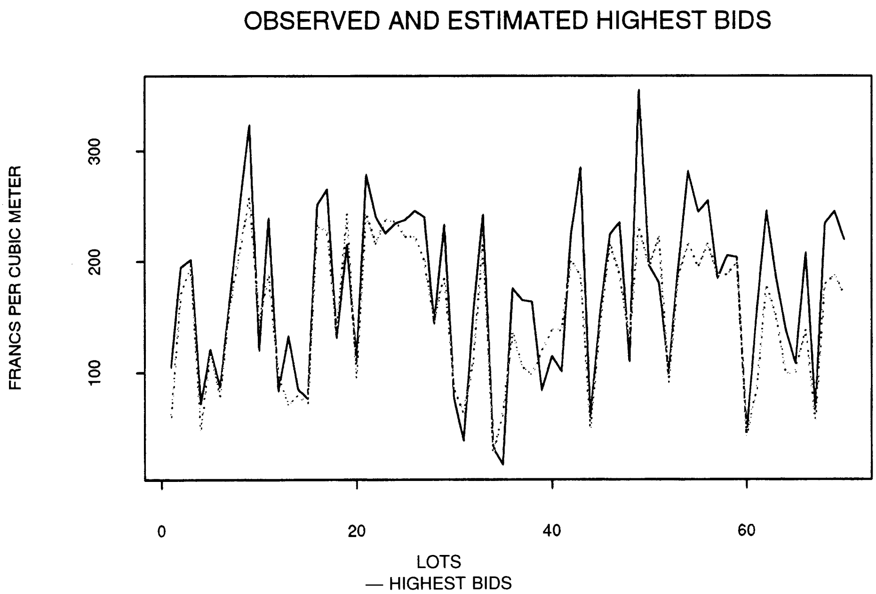

FIGURE 7

Observed and Estimated Highest Bids.

derive from switching to an optimal policy of public reservation prices (see (7)). For both types of auctions the expected utility is of the form

$$\sum_{l=1}^{L} q_l \, E_{v_{0l}} \left[v_{0l} \, Pr \left[s_l(y) < p_l(v_{0l}) \right] + \int_{\{y: s_l(y) \geqq p_l(v_{0l})\}} s_l(y) \, dF_{l\,\max}(y) \right],$$

where $s_l(\cdot)$ and $p_l(\cdot)$ are the relevant strategies and q_l is the volume of timber of auction l.

We can approximate these quantities by

$$\sum_{l=1}^{L} q_l \left[v_{0l} \int_0^{s_l^{-1}(p_l(v_{0l}))} dF_{l\,\max}(y) + \int_{s_l^{-1}(p_l(v_{0l}))} s_l(y) \, dF_{l\,\max}(y) \right],$$

where

- q_l is the observed volume of timber in auction l,
- v_{0l} is the seller's private value in auction l, which is equal to his observed reserved price p_{0l}.
- $s_l(\cdot)$, $p_l(\cdot)$, and $F_{l\,\max}$ are respectively the estimated strategies of the buyer and the seller and the estimated distribution of the maximal valuation of the sellers for auction l.

Using this formula, we obtain 14 019 341 FF for a secret reservation price instead of 14 459 681 FF for a public reservation price. That is, we obtain a gain for sellers of 440 340 FF from moving to a public reservation price.

To conclude this paper, we will offer a conjecture for explaining the choice of secret reservation prices despite. Proposition 1 and the preceding numerical results, which illustrate it.

The expected *sales* for sellers (i.e., excluding the sellers' private values) are defined for both auctions by

$$\sum_{l=1}^{L} q_l \, E_{v_{0l}} \left[\int_{\{y: s_l(y) \geqq p_l(v_{0l})\}} s_l(y) \, dF_{l\,\max}(y) \right],$$

and approximated by

$$\sum_{l=1}^{L} q_l \left[\int_{s_l^{-1}(p_l(v_{0l}))} s_l(y) \, dF_{l\,\max}(y) \right].$$

Using this formula, we obtain 12 785 974 FF for a secret reservation price instead of 10 520 137 FF for the public reservation price.

If this superiority in expected sales of the secret reservation price was general, then this might give a plausible explanation for the frequent use of auctions with secret reservation price since the organizers of the auctions are paid a percentage of sales. For a 4% commission the gain for the organizers from a secret reservation price is of the order of 88 000 FF in our case.

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