JOURNAL OF THEORETICAL POLITICS

*Article*

# A model of electoral alliances in highly fragmented party systems

Journal of Theoretical Politics
2021, Vol. 33(1) 3–24

DOI: 10.1177/0951629820963182
journals.sagepub.com/home/jtp
SAGE

**Marcelo de C Griebeler**
Federal University of Rio Grande do Sul, Brazil

**Roberta Carnelos Resende**
UNISINOS, Brazil

## Abstract

Highly fragmented party systems are composed of a large number of political groups, many of which have negligible bargaining power when it comes to negotiations of electoral alliances. Inspired by the Brazilian system—one of the most fragmented in the world—we provide a stylized model of electoral alliances in which there is a competition for the support of the small parties. Two leading parties try to entice a small one by simultaneously offering transfers (e.g. government positions, support in other elections, prestige). Through a first-price sealed-bid auction approach, we can analyze the role of two particular factors in alliance formation, namely pragmatism and ideology. Our findings show that, in equilibrium, the favorite party tends to offer lower transfers than the underdog. Furthermore, the closer the leading and the small parties are in terms of ideology, the smaller the transferred amount. When the ideological closeness between the small party and the underdog is sufficiently large—relative to the favorite—the effect may be strong enough to overcome the electoral advantage of the favorite.

## Keywords

Auctions; elections; political economy

**Corresponding author:**
Marcelo de C Griebeler, Universidade Federal do Rio Grande do Sul – UFRGS, Faculdade de Ciências Econômicas, Departamento Economia e Relações Internacionais, Avenida João Pessoa 52, Centro Histórico, Porto Alegre -RS 90040-000, Brazil.
Email: marcelo.griebeler@ufrgs.br

## 1. Introduction

During the Brazilian presidential campaign of 2002, the domestic political scene was surprised by the unlikely alliance between the Workers' Party (PT) and the Liberal Party (PL).[1] While PT was one of the largest left-wing Brazilian parties, with a history highly identified with popular movements and known by its anticapitalism speech, PL was a small right-wing political group founded by businessmen and strongly associated with, for example, the defense of taxes cuts.[2] The alliance, which contained three other left-wing parties (PCdoB, PCB, and PMN[3]), chose Luiz Inácio Lula da Silva (PT) and José Alencar (PL) as presidential and vice-presidential candidates, respectively. Despite the ideological differences between the two main parties, the alliance succeeded and won the election, receiving 61.27% of the valid cast votes. The great success of the alliance made PT and PL continue the cooperation in the following election in 2006, when the same candidates were reelected.[4]

The examples of the aforementioned presidential elections are far from being an exception in the Brazilian political scenario. The indifference regarding ideology is even more common in campaigns for offices at the regional level (state and municipal), where well-known political enemies often ally with each other with the objective of winning. Consider, for instance, the alliance between the Party of the Liberal Front (PFL, currently Democrats, DEM), a political group traditionally viewed as a symbol of the national right wing, and PT in the elections for governor in 2014 in the states of Maranhão, Pará, and Paraíba. As an example, at a local level, in 2008, the Party of the Brazilian Social Democracy (PSDB), a traditional opponent of PT in presidential elections, allied with the latter in more than 20% of the Brazilian municipalities. Finally, consider the alliance between PT and the Progressive Party (PP)—which, despite the name, is one of the largest right-wing national parties—in the race for the mayor of the city of São Paulo in 2012, the largest Brazilian electoral college.

These examples may suggest that electoral alliances are always formed pragmatically, with the only objective of winning the election.[5] In fact, the majority of the literature on Brazilian electoral alliances finds that pragmatism is one of the main reason why parties choose to join and form a coalition (Borges et al., 2017; de Lima Júnior, 1983; Lavareda, 1991; Machado, 2012; Miguel and Machado, 2007; Soares, 1964).[6] However, although pragmatism seems to be the strongest driving force behind the alliance decision-making process,[7] empirical evidence has reported that, in some contexts, ideology does have an important role in the choice of whom to ally with (Miranda, 2013; Oliveira, 1973; Santos, 1987; Krause and Godoi, 2010; Schmitt, 1999).

In this paper, we build a model in which political parties have to decide whom to ally with in order to win an election for an executive office. Inspired by the Brazilian scenario, we highlight the role of two particular factors in the decision of allying with another party,[8] namely pragmatism and ideology. Instead of investigating the determinants of the formation of electoral alliances—or explaining their formation—our model studies how the two different factors (determinants)

aforementioned affect the terms of the deals—and, by extension, the probability of the different alliances forming. We assume that leading parties try to entice small ones by simultaneously offering transfers (e.g. government positions, support in other elections, prestige, power of being part of the winning alliance). Thus, one can think of the electoral alliance game as a sealed-bid first-price auction, in which the leading parties are the bidders and the small one is the object being auctioned. To explain the channels through which the two factors affect parties' decisions, we explore the role of each one separately.

In a static setting, we first provide a baseline model that allows us to analyze (pure) pragmatism. Two leading parties play a game in which each of them makes offers to a small one to convince it to join them in an electoral alliance. Given that the probability of victory of every possible alliance is common knowledge, parties are able to calculate their expected payoffs and thus the small party chooses to ally with the one that offers the highest expected transfer. Our findings show that the party that is favorite in the campaign tends to offer fewer transfers than the underdog. However, it is not always the case that the favorite entices the small one since leading parties can value the office differently. We also show that the closer the leading and small parties are in terms of ideology the smaller the transferred amount. In fact, when the ideological closeness between the small party and the underdog is sufficiently large—relative to the favorite—the effect may be strong enough to overcome the electoral disadvantage.

Our main contribution is to provide a stylized model that is suitable for analyzing alliances in societies where party systems are highly fragmented, such as Brazil. In such societies, there is a large number of parties, which may have different sizes in terms of electorate or seats in Congress. In most cases, this difference results from a very uneven distribution of political power across parties. The consequence is that a large share of the political groups is small enough *and thus unable to* affect presidential elections individually. Our main assumption is that these small parties may be seen as 'price-takers', in the sense that their bargaining power is null when it comes to alliance agreements: they receive 'take-it-or-leave-it' offers from the leading parties, rather than bargaining with them. This implies that small parties may be interpreted as objects of auctions, whose bidders are the leading parties.

It is worth noticing that our setup precludes parliamentary systems in which such small parties might well be expected to compete independently and be in a stronger bargaining position (as far as entering government) after the election. In fact, to be applied, the model we develop here requires not only high fragmentation but also that the system be presidential. As we show, it is not optimal for the small party to run against the leading ones. Given that there is no chance of winning the election independently, and the winner makes transfers only to members of its coalition, it is always preferable to join a leading party. For the Brazilian case, for example, Borges and Peron (2018) find that those who receive government positions and support in other elections are, in general, only the members of the pre-electoral coalition—there are no transfers of this sort to post-electoral allies.

Among the reasons why our contribution is novel is the lack of models able to explain realistically how pre-electoral alliances are formed in highly fragmented

systems. In fact, the scarce literature on electoral alliances (e.g. Bandyopadhyay et al., 2011; Blais and Indridason, 2007; Golder, 2006a) has understood this process as a result of bargaining between political parties. That approach, however, assumes that all players have some bargaining power, which may not be the case when there is high fragmentation. Moreover, bargaining models neglect the competitive aspect of political alliances. In multiparty systems—and in those that are highly fragmented, in particular—the support of a given party may be the object of competition between other political groups. While in an auction framework the analysis of this competition is straightforward, bargaining models would require a complex setting to do the same, with small parties receiving offers and making counteroffers to more than one leading party simultaneously.

But how can one know whether a system is fragmented enough to be better analyzed by our auction model? In other words, to apply our approach to a given country, first it is necessary to measure the share of parties that are weak enough to be considered 'small'. One way to do this is by calculating the so-called effective number of parties (ENP) (Golosov, 2010; Laakso and Taagepera, 1979), which takes into account the relative size of those political groups. By comparing the ENP with the actual number of parties ($N$), we have a proxy of the fragmentation of the system as well as the strength of its members. As the literature has shown (van de Wardt, 2017), low ratios between ENP and $N$—high fragmentation—are, in general, associated with some electoral institutions, such as proportional representation. In particular, so-called coalitional presidentialism, a form of politics in which presidents who lack single-party legislative majorities build and manage cross-party support in legislative assemblies (Chaisty et al., 2018), is largely correlated with high fragmentation.

Brazil is a textbook example of coalitional presidentialism.[9] Its peculiar political system, which combines two different approaches, is the main reason for the high fragmentation and low ENP/$N$.[10] Executives and senators are elected through a majoritarian electoral system: presidents, governors, and mayors win by majority runoff, while senators are elected by a plurality of the vote. However, both state and federal deputies are elected according to a system of open-list proportional representation: seats are awarded in proportion to the votes that each coalition wins; that is, the candidates who win seats are those who win the most votes within each coalition.

In addition to proliferating the number of (mostly powerless) parties, the Brazilian system makes the formation of alliances beneficial for both leading and small parties. On the one hand, leading parties want to entice the small ones in order to obtain more votes for executive elections. On the other hand, small parties benefit from being part of an alliance by increasing their chances of having an elected deputy.[11] In fact, Brazil presents several of the institutional characteristics that Spoon and West (2015) (see also Ferrara and Herron, 2005) found as encouraging of pre-electoral coalitions in races for the presidency (e.g. strong presidential powers and plurality rule).

Finally, we choose to analyze bilateral rather than bloc alliances for several reasons. First, leading parties often have enough power to negotiate with small ones

individually, after analyzing whether it is worth trying to entice each one of them. Thus, bilateral alliances may bring intuition about the microeconomics of the phenomenon, which may be absent when decisions are made by blocs of parties. Second, the framework suitable to study bloc alliances is very similar to the one used in the literature about post-electoral coalitions, which is already well-developed (Bandyopadhyay et al., 2011; Machado, 2009, 2012; Riker, 1962). The analysis of bilateral alliances, however, is still very incipient, such that our model may be an important contribution to this field. Third, although those negotiations are often considered as 'black boxes', the recent literature has suggested that this may be the way alliances are formed in countries such as Brazil (Borges and Peron, 2018).

## 1.1. Related literature

Although there are many examples that would motivate a detailed analysis of the strategic nature of electoral alliances, the literature on the subject is scarce. Much of the research in this field draws on the studies of Sona Golder, who provides the first—and one of very few, to the best of our knowledge—analytical model of pre-electoral coalitions that takes into account the strategic aspects of the decision (Golder, 2006a). The bargaining game proposed in her seminal book considers two parties (or two party leaders) who must decide whether to form an electoral alliance or not. Among other findings, her study shows that electoral coalitions are more likely when the potential allies are ideologically close and when there is an extreme opposition to the alliance. Bandyopadhyay et al. (2011) study a game-theoretic model where three parties can form coalitions before the election. However, their paper also investigates how these alliances can affect the coalitions formed after the election. A similar model is proposed by Blais and Indridason (2007), where the main interest is in the formation of electoral pacts in majority runoff, with special attention to the consequences of the presence of extremist parties.

While the three studies aforementioned have some similarities to ours, that of Bandyopadhyay et al. (2011) is the closest, as their model investigates the possible motives for pre-electoral agreements. Nevertheless, as we have seen, there are still important differences between their approach and ours. For example, we use an auction framework instead of bargaining (Bandyopadhyay et al., 2011; Blais and Indridason, 2007; Golder, 2006a), and we consider the case of two main parties competing for the support of a small one, instead of only two (Blais and Indridason, 2007; Golder, 2006a) or three (Bandyopadhyay et al., 2011) main parties running in an election and possibly allying with each other. Moreover, the three theoretical models discussed analyze legislative elections rather than presidential ones, which is our main interest.

Some of the main empirical contributions to the field are also from Sona Golder. The two major existing potential explanations for electoral coalitions (mainly for parliamentary elections), namely, disproportionality and the signaling hypothesis, are tested by Golder (2005, 2006a) for 23 parliamentary democracies from 1946 to 2002. While the first explanation states that disproportional electoral

rules increase the likelihood of alliances, the latter has three variants, which say that the alliances are devices to signal to voters: (i) that member parties can form an effective government in the case of victory; (ii) the identity of potential future government coalitions; and (iii) the alliance's desire to have voters play a larger role in determining government coalitions. While evidence is found that supports the disproportionality hypothesis, the same does not occur for the signaling one. Our model takes such evidence into account by assuming that parties (or party leaders) do not care about voters, such that they do not intend to signal anything by forming alliances. Instead, they only care about winning (pragmatism) and ideology.

The empirical literature has focused more on electoral alliances in legislative elections. In particular, most of the studies investigate how the political and institutional environment affects the formation of pre-electoral pacts (see e.g. Blais and Indridason, 2007; Ferrara and Herron, 2005). A descriptive and historical approach is used in some papers, such as that of Golder (2006b), who studies electoral coalitions in the French Fifth Republic and post-1987 South Korea. Other studies focus on specific alliances in a given country, such as that of Spoon and Gómez (2017) for the case of the Partido Revolucionario Institucional (PRI) and the Partido Verde Ecologista de México (PVEM) in Mexico.[12] Electoral alliances in presidential elections, in turn, are analyzed, for example, by Spoon and West (2015) and Kellam (2017). Although both use data from South America countries (Spoon and West (2015) uses data from some Europe countries as well), their interest is different: while the former investigates the effects of institutional environment, the latter focus on the characteristics and goals of the allies. In the Brazilian context, Soares (1964) and Melo (2015) provide an analysis focusing on historic aspects rather than strategic ones. There are also papers that try to explore possible reasons for forming alliances (pragmatism, ideology, or loyalty), but all those attempts do not provide any formal analysis or game-theoretical approach (Krause and Godoi, 2010; Resende and Epitácio, 2017). In addition, with the exception of Resende and Epitácio (2017), the literature focuses only on bloc coalitions.

### *1.2. Outline*

This paper is organized as follows. The next section presents our baseline model, in which we explore the role of pragmatism in driving the electoral alliance decision. Our framework, composed of two leading parties and one small party, also allows us to analyze the influence of ideology on the parties' alliances. The strength of our main assumption, namely that bilateral political alliances in highly fragmented systems can be properly modeled through auctions, is discussed in Section 3. Finally, in Section 4, which concludes the paper, some extensions are suggested.

## 2. Baseline model: political pragmatism

Consider a society where there exist only three political parties, $A$, $B$, and $C$. There will be an election in the next period and the parties must choose which alliances

they will form before it takes place. None of them individually has the absolute majority of votes; an alliance may be essential to assuring the victory. Without loss of generality, we assume that party $A$ is the favorite and party $B$ is the underdog. Party $C$ is a small party whose electoral support will be the object of competition between the other two, such that the only two alliances we allow are $A$ and $C$ (henceforth, $AC$), and $B$ and $C$ (henceforth, $BC$). We can justify such an assumption by thinking that only the two largest parties have charismatic leaders and thus they are the only parties able to run in the election as the leaders of their alliances. Observe that, because there are only two potential candidates in the baseline model—party $C$ never runs alone, as we will see—it applies to both majority and plurality rules. It would also apply to the second round of a runoff system if we assumed that the three parties run in the first round but only $A$ and $B$ proceed to a second one.

Let $N_i$ be the share (base) of votes for party $i = A, B, C$; then these assumptions can formally be expressed as $N_A + N_B + N_C = 1$, $1/2 > N_A > N_B > N_C$, and $N_A < N_B + N_C$. This latter inequality means that even though $B$ is the underdog, it wins the election if it succeeds in obtaining all votes of $C$. However, following Blais and Indridason (2007), we assume that the proportion of voters that follows $C$ in its alliance with another party is a random variable $\mu$.[13] The remaining party $C$ voters—the share $1 - \mu$—do not cast a ballot in the election (or their votes are invalid).[14] One possible reason why part of the base of $C$ does not follow it and thus does not vote for the alliance is ideological: they are loyal only to the values of party $C$ and therefore do not vote for another political group.

This assumption creates uncertainty about the election results: if $B$ and $C$ form an alliance, their total share of votes is $N_B + \mu N_C$, which can be less than, greater than, or equal to $N_A$. Yet it is straightforward to see that there exists $\mu^* \in (0, 1)$ such that $N_B + \mu^* N_C = N_A$, which implies that $BC$ wins the election if and only if $\mu > \mu^*$.[15] For future use, observe that $\mu^* = (N_A - N_B)/N_C$. We assume that all this information is common knowledge. In particular, the base of each party $i$, $N_i$, is public information.[16] Observe that, while the share of $C$ voters who are loyal to the party is uncertain and, in general, less than the total, all voters of both parties $A$ and $B$ are loyal—and that this is common knowledge. Although this assumption can be easily relaxed and potentially change the results (see the discussion after Proposition 1), we choose to include only a single source of uncertainty over the electoral outcome—and therefore only a single random variable. The next assumption establishes the distribution of $\mu$, which we also assume to be common knowledge.

**Assumption 1.** $\mu$ is a continuous uniform random variable on $[0, 1]$

By assuming that $\mu$ is uniformly distributed, we can easily obtain the probability of victory of each possible alliance[17]

$$Pr(\text{AC wins}) = Pr(N_A + \mu N_C \geqslant N_B) = Pr\left(\mu \geqslant \frac{N_B - N_A}{N_C}\right) = 1$$

$$Pr(\text{B wins}) = Pr(N_A + \mu N_C < N_B) = Pr\left(\mu \leq \frac{N_B - N_A}{N_C}\right) = 0$$

$$Pr(\text{BC wins}) = Pr(N_B + \mu N_C > N_A) = Pr\left(\mu \geqslant \frac{N_A - N_B}{N_C}\right) = 1 - \frac{N_A - N_B}{N_C} = 1 - \mu^*$$

$$Pr(\text{A wins}) = Pr(N_A \geqslant N_B + \mu N_C) = Pr\left(\mu \leq \frac{N_A - N_B}{N_C}\right) = \mu^*$$

Observe that we do not consider the possibility of $C$ being neutral and not forming an alliance with any other party. In fact, as we will see, the payoff function of $C$ implies that choosing to form an alliance is always preferable to being neutral.

An important remark is that our model does not consider the presence of swing voters. Each voter belongs to a base of some party; that is, he is assumed to be a supporter. In addition, voters are not influenced by the platforms announced by candidates. In fact, the only decision made by parties is whether or not to ally with another political group, such that there is no announcement aiming to influence voters' choice. Our model, therefore, assumes that the political base of each party is fixed. While such an assumption allows us to focus on the transfers that a leading party might offer to a small one, it precludes an investigation about changes in the platforms as a mechanism to attract potential electoral allies.[18]

## 2.1. Timing of the game

The electoral alliance game follows the following steps:

1. The two leading parties try to attract party $C$ to their alliances by making simultaneous offers to it. Those offers are monetary and non-monetary transfers, as we present.
2. Party $C$ accepts the offer that yields the highest expected payoff.
3. The election takes place and the party (or alliance) with the highest number of votes cast wins.
4. The winning leading party receives its payoffs and, in the case of an alliance, transfers what was promised to party $C$.

The game is very similar to a first-price sealed-bid auction, given that the two parties make their 'bids' simultaneously and the one whose bid is the highest is the 'winner', that is, the one that entices party $C$. In the next section, we present the parties' payoffs and analyze the game's equilibrium.

## 2.2. Players

*2.2.1. Party C.* The payoff of party $C$ depends only on the transfers it receives from the alliance's leading party ($A$ or $B$). These transfers can include government positions, support in other elections (e.g. at state and local levels), prestige and power of being part of the winning alliance, and learning some new electoral technology from the leading party, among others. In fact, it is very commonly the case that governments allocate positions such as ministries among electoral allies as a reward

for their support in previous elections.[19] There is no negotiation between parties; that is, the offer is made by the leading party to $C$ and the only action the latter takes is accepting or not. We assume that when the parties announce their offers, before the beginning of the campaign, they are able to commit to carrying out their promises if elected.[20]

Formally, we assume that party $C$ is risk-neutral and thus its Bernoulli utility function is given by $v(t^i) = t^i$, where $t^i$ is the transfer from party $i$. Notice that by staying neutral its payoff is null, which implies that it always chooses to form an alliance. Thus, party $C$ chooses to ally with $A$ if and only if

$$t^A \geqslant t^B(1 - \mu^*)$$

Given that the election results are uncertain when the alliance $BC$ is formed, the expected payoff of party $C$, in this case, must take into account the probability of victory of $BC$. Moreover, if that alliance is defeated, its payoff is zero because now there are no transfers to be made. An initial conclusion that can be drawn from the behavior of party $C$ is that the higher the advantage of the favorite party, the higher must be the transfers the underdog offers to $C$ to attract it to the alliance.

*2.2.2. Parties A and B.* Parties $A$ and $B$ are also risk-neutral and maximize their 'net rent', which is the utility of holding office $R^i > 0$ minus the transfers $t^i$ made to party $C$ in the case of alliance, for $i = A, B$. As usual, the utility from being in office may be derived from salary and other monetary compensations but may also be due to ego rents, which include prestige, power, and other psychological rewards associated with social status and political influence.

The expected utility of party $A$ is therefore

$$U^A = Pr\left(t^A \geqslant t^B(1 - \mu^*)\right)\left(R^A - t^A\right) + Pr\left(t^A < t^B(1 - \mu^*)\right)\mu^* R^A$$

while the expected utility of party $B$ is

$$U^B = Pr\left(t^A < t^B(1 - \mu^*)\right)(1 - \mu^*)\left(R^B - t^B\right)$$

Observe that we use the implicit assumption that whenever the party loses the election its utility is null, since $R^i = 0$ and there is no transfer to be made, that is, $t^i = 0$.

Their payoffs take into account all possible results. First, they consider the possibility of alliance $AC$, which occurs with probability $Pr(t^A \geqslant t^B(1 - \mu^*))$. In this case, $AC$ wins—and consequently $B$ loses—with certainty. Payoffs of party $A$ and party $B$ are $R^A - t^A$ and zero, respectively. The second possible result is the alliance $BC$, which occurs with probability $Pr(t^A < t^B(1 - \mu^*))$. Now, the probability of victory of $A$ is $\mu^*$, while that of the alliance is $1 - \mu^*$. In the case of victory of $A$, there is no transfer to be made, such that its payoff is $R^A$, while if $BC$ wins, the party $B$ has a net rent of $R^B - t^B$.

## 2.3. Equilibrium

Given the first-price auction framework of our model, we look for a symmetric Nash Bayesian equilibrium. To do so, we restrict our attention to the case where the 'values' that parties $A$ and $B$ assign to victory in the election, namely $R^i$, are private information. In other words, party $i$ knows its own $R^i$ and only the distribution of the value of the other party $j \neq i$, which is assumed to be common knowledge information. We detail this assumption next.

**Assumption 2.** $R^i$ is a random variable independently and identically distributed on $[0, \bar{R}]$, according to a continuous uniform distribution, for $i = A, B$.

A well-known result in auction theory is that when a player's valuations are uniformly distributed, a linear equilibrium exists and is unique. Thus, Assumption 2 allows us to focus on strategies with the functional forms $t^A = a^A + b^A R^A$ and $t^B = a^B + b^B R^B$, where $a^i, b^i$, for $i = A, B$, are parameters to be determined. It is reasonable to expect that $b^i > 0$ because the larger the utility from being in office the party gets the higher its offer to $C$ must be.

By substituting the linear strategy of party $B$ into the payoff of party $A$, we have the following maximization problem for the latter

$$\max_{t^A} \frac{1}{\bar{R}} \left[ \frac{t^A - a^B(1-\mu^*)}{(1-\mu^*)b^B} \right] \left(R^A - t^A\right) + \left\{ 1 - \frac{1}{\bar{R}} \left[ \frac{t^A - a^B(1-\mu^*)}{(1-\mu^*)b^B} \right] \right\} \mu^* R^A$$

where the probabilities were calculated using Assumption 2. The first-order condition (FOC) is given by

$$-\frac{1}{\bar{R}} \left[ \frac{t^A - a^B(1-\mu^*)}{(1-\mu^*)b^B} \right] + \frac{R^A - t^A}{\bar{R}(1-\mu^*)b^B} - \frac{\mu^* R^A}{\bar{R}(1-\mu^*)b^B} = 0 \tag{1}$$

which has as solution

$$t^A = (1-\mu^*) \frac{(R^A + a^B)}{2} \tag{2}$$

One can see in equation (1) the trade-off that party $A$ faces. On the one hand, by increasing $t^A$, its chance of enticing $C$ increases, which makes its victory more likely. On the other hand, higher transfers decrease its disposable rent. Which of those two effects dominates depends primarily on the probability of $A$ winning without the support of $C$. Observe in equation (2) that, regardless of the value of $a^B$, the greater $\mu^*$ the less $t^A$. In fact, its transfer is null when there is certainty of victory ($\mu^* = 1$).

A similar procedure for party $B$ yields

$$\max_{t^B} \frac{1}{\bar{R}} \left[ \frac{t^B(1-\mu^*) - a^A}{b^A} \right] (1-\mu^*) \left(R^B - t^B\right)$$

whose FOC is

$$-\frac{1}{\bar{R}}\left[\frac{t^B(1-\mu^*)-a^A}{b^A}\right](1-\mu^*)+\frac{(1-\mu^*)^2(R^B-t^B)}{\bar{R}b^A}=0 \tag{3}$$

such that the solution is

$$t^B=\frac{R^B}{2}+\frac{a^A}{2(1-\mu^*)} \tag{4}$$

The incentives for party $B$ to increase or decrease the transfers it makes to party $C$ are similar to those of its opponent. Higher transfers make the alliance $BC$ more likely and thus increase its chances of victory but at the same time they decrease its net rent $R^B - t^B$. However, the marginal effect of one unit of transfers on the probability of victory is larger for party $B$ than for party $A$, given that even with the alliance $BC$ there is no guarantee of success, and without it the defeat is sure. Thus, it is expected that, in equilibrium, $B$ has more incentives to transfer than $A$.

The parameter values can be obtained as a solution of the system $t^A = a^A + b^A R^A$, $t^B = a^B + b^B R^B$, and equations (2) and (4). It is straightforward to show that $a^A = a^B = 0$, $b^A = (1-\mu^*)/2$, and $b^B = 1/2$. The game's equilibrium is presented in the following proposition.

**Proposition 1.** *The only Bayesian Nash equilibrium of the electoral alliance game given previously is*

$$t^A=\frac{(1-\mu^*)}{2}R^A \tag{5}$$

$$t^B=\frac{R^B}{2} \tag{6}$$

*In other words, the underdog party offers party C a transfer that is half of its utility of being in office, while the favorite makes an offer that is less than half of its valuation.*

These results are somehow expected, since we have already identified that the electoral alliance game is very similar to a first-price auction. A well-known result of auction theory (Krishna, 2009) is that, under almost the same conditions our model presents (Assumption 2 in particular), in equilibrium, each bidder chooses to bid half of his valuation in a first-price auction. Therefore, our result is different from that only because the favorite party presents a discount factor of $1-\mu^*<1$. Such a discount is higher when the degree of favoritism of party $A$ is higher. In particular, when it wins with certainty in any situation, there is no incentive for it to transfer to $C$.[21]

The intuition behind the equilibrium described in Proposition 1 is simple. Everything else held constant, an alliance with $A$ is always preferable to one with $B$ for party $C$ because of the certainty of victory. The two leading parties anticipate this behavior, such that party $A$ is aware of its advantage. In fact, an analysis of equations (1) and (3) shows that $A$ can obtain the same probability of victory as $B$ offering less than half of its valuation. The difference in their offers may also be

visualized when one supposes that $R^A = R^B$: in this case, the amount offered by the favorite is less than the underdog's offer. Finally, it is also expected that the magnitude of its advantage—measured by $\mu^*$—has an impact on its optimal transfer, namely, it must be the case that the higher $\mu^*$ the lower $t^A$, which can easily be confirmed in equation (5).

We can perform a welfare analysis of the equilibrium from the perspective of party $C$. Given the probability of victory of each possible alliance, in equilibrium, its expected payoffs are $t^A = (1 - \mu^*)R^A/2$ and $t^B(1 - \mu^*) = (1 - \mu^*)R^B/2$ when it allies with $A$ and $B$, respectively. This implies that party $C$ will be better off and will thus choose to ally with the leading party whose $R$ is the highest. Under the assumptions of our model, therefore, parties that get more utility in the case of victory tend to form pre-electoral alliances. Assume that corrupt parties get more utility of holding office—they appropriate public money in addition to the usual benefits of being in power. Then an implication of this result is that this type of party will have a higher probability of forming pre-electoral alliances.

One interesting implication of the equilibrium described here is that it is possible to interpret it as a violation of Gamson's law (proportionality norm). In fact, if we assume that the size of the electoral base $N_i$ is proportional to the number of seats (relative size) that each party $i$ contributes to the government's legislative majority, then a junior coalition partner should receive a small share of the cabinet portfolio and government perks. Our model, however, predicts that, in equilibrium, the small party may be able to extract up to half of the leading party's perk. It is worth noticing that this is out of step with empirical regularity, as recent work in this area points out that the office bonus for small parties is particularly likely in parliamentary regimes, whereas presidential regimes are more likely to give the president's party an advantage.

One can understand this result by analyzing, in detail, the definition of the variable $R$. If it includes not only government positions and cabinet portfolio but also psychological benefits, such as ego rents (e.g. private enjoyment of prestige and power), then $R/2$ is not necessarily half of the government's perk. Furthermore, in clientelistic systems, such as Brazil, many positions are often handed out that are not cabinet portfolios (e.g. higher offices in state-owned companies). In fact, Batista (2018) shows that the type of transfers that the leading party makes to small ones may include policies, offices, and budgets controlled by the ministry in question. Under this interpretation, therefore, Gamson's law is not necessarily violated.

Nevertheless, if we interpret the variable $R$ as not including any other sort of benefit, Proposition 1 does violate the proportionality norm. In this case, it is important to recall that there is a competition between leading parties for the support of a small one, instead of a bargaining between partners with (potentially) different but positive power—the way in which previous researchers have studied the problem. As the alliance with party $C$ is the only chance that party $B$ has of winning, its actual bargaining power is higher than it would be if there were an actual bargaining process between them. Moreover, $R/2$ is the upper bound of the transfer, such that if the alliance is with $A$ the value is lower. In fact, if $\mu^*$ is close to

one, such a transfer is close to zero. These differences may explain the non-validity of the proportionality norm and thus corroborate the findings in the experimental literature (Fréchette et al., 2005), in which non-cooperative games may violate Gamson's law.

## 2.4. The influence of ideology

Let us now relax the assumption that the share of party $C$ voters that follows it in an alliance does not depend on the specific ally. Instead, assume that when $C$ chooses to ally with $A$, the share $\mu_A$ of its voters votes for $AC$. Similarly, let $\mu_B$ be the share of $N_C$ that votes for $BC$ in the case of an alliance. One reason why $\mu_A$ and $\mu_B$ may be different is the ideological closeness between the voters of party $C$ and those of the potential allies. The idea is that the closer the voters of two parties are in terms of ideology the larger the share of them that votes for a potential alliance.

We model the potential heterogeneity between the ideologies of the parties—and of their voters—by assuming that the supports of $\mu_A$ and $\mu_B$ depend on a parameter $\lambda \in (0, 1)$, which measures the relative ideological closeness between parties $B$ and $C$. Thus, the higher $\lambda$, the closer the ideologies of the voters of party $B$ and party $C$, compared with the proximity between the ideologies of the voters of parties $A$ and $C$. This assumption may be seen as an unconventional way of capturing ideological proximity, especially in light of the fact that the baseline model does not account for an ideological policy pact along the contract curve of two negotiating parties.[22] However, if we include the benefits of having parties' preferred policies implemented (e.g. psychological ones) in $R$, the terms of the deal may contain some policies preferred by the small party that will be implemented by the leading one (in the case of victory). Although it implies that, in this case, the baseline case can no longer be called 'pure pragmatism', this change makes Assumption 3 similar to most of the literature. We explain the assumption of ideological proximity in detail next.

**Assumption 3.** $\mu_A$ and $\mu_B$ are continuous uniform random variables on $[0, 1-\lambda]$ and $[\lambda, 1]$, respectively, where $\lambda \in (0, 1)$. Moreover, $\mu_A$ and $\mu_B$ are independent.

Observe that the higher the $\lambda$ the more likely the realization of high values (close to one) of the random variable $\mu_B$ and the realization of low values (close to zero) of $\mu_A$. The impact of the ideological closeness on the probabilities of victory can be seen as

$$Pr(N_A + \mu_A N_C \geqslant N_B) = 1$$

$$Pr(N_A + \mu_A N_C < N_B) = 0$$

$$Pr(N_A \geqslant N_B + \mu_B N_C) = Pr(\mu_B \leq \mu^*) = \frac{\mu^* - \lambda}{1-\lambda}$$

$$Pr(N_A < N_B + \mu_B N_C) = 1 - \frac{\mu^* - \lambda}{1-\lambda} = \frac{1-\mu^*}{1-\lambda}$$

Notice that if $\lambda \geqslant \mu^*$, then $Pr(N_A \geqslant N_B + \mu_B N_C) = 0$ and $Pr(N_A < N_B + \mu_B N_C) = 1$, such that we would have a very simple strategic interaction. In fact, in this case, any leading party able to entice party $C$ assures its victory in the election. The underlying game would be a standard first-price auction and the equilibrium would satisfy $t^i = R^i/2$ for $i = A, B$.

The more interesting case is when $\lambda < \mu^*$. Now, party $C$ chooses to ally with $A$ if and only if $t^A(1-\lambda) \geqslant t^B(1-\mu^*)$. By using Assumption 2, we can once again focus on linear strategies. The same procedure used in the baseline case allows us to write the optimization problem of party $A$ as

$$\max_{t^A} \frac{1}{\overline{R}} \left[ \frac{t^A(1-\lambda) - a^B(1-\mu^*)}{(1-\mu^*)b^B} \right] \left(R^A - t^A\right) + \left\{ 1 - \frac{1}{\overline{R}} \left[ \frac{t^A(1-\lambda) - a^B(1-\mu^*)}{(1-\mu^*)b^B} \right] \right\} \frac{(\mu^* - \lambda)}{1-\lambda} R^A$$

whose solution is given by

$$t^A = \frac{(1-\mu^*)}{2(1-\lambda)} \left(a^B + R^A\right) \tag{7}$$

Likewise, party $B$ solves the following program

$$\max_{t^B} \frac{1}{\overline{R}} \left[ \frac{t^B(1-\mu^*) - a^A(1-\lambda)}{(1-\lambda)b^A} \right] \frac{(1-\mu^*)}{(1-\lambda)} \left(R^B - t^B\right)$$

whose solution is given by

$$t^B = \frac{a^A(1-\lambda)}{2(1-\mu^*)} + \frac{R^B}{2} \tag{8}$$

The optimization problem of both parties is the same as in the baseline case except for the presence of the parameter $\lambda$. One can see that as the ideological closeness between voters of parties $B$ and $C$ increases, the expected payoff of party $A$ decreases while the opposite happens with the payoff of $B$. As we will see, this will have an impact on the equilibrium transfers. The equilibrium is once again obtained by solving the system with $t^A = a^A + b^A R^A$, $t^B = a^B + b^B R^B$, and equations (7) and (8). The next proposition analyzes the two possible cases.

**Proposition 2.** *Consider the electoral alliance game with ideology. If* $\lambda \in (0, \mu^*)$*, then the only Bayesian Nash equilibrium is*

$$t^A = \frac{(1-\mu^*)}{2(1-\lambda)} R^A$$

$$t^B = \frac{R^B}{2}$$

*If* $\lambda \geqslant \mu^*$*, then the only Bayesian Nash equilibrium is* $t^i = R^i/2$ *for* $i = A, B$.

We have already analyzed the trivial case in which $\lambda \geqslant \mu^*$. Suppose then that $\lambda \in (0, \mu^*)$ and compare the results with those of Proposition 1. The only difference is the presence of the parameter representing the ideological closeness in the optimal choice of party $A$. In fact, we can observe that the marginal impact of $\lambda$ on $t^A$ is positive while there is no effect on $t^B$. This means that when the ideological views of voters of parties $B$ and $C$ are closer, the ex-ante electoral advantage of party $A$ is lower. The channel through which this effect operates is the higher probability of high realized values of $\mu_B$. Finally, given the assumption of $\lambda < \mu^*$, it is always the case that $t^A < R^A/2$. In other words, even with the presence of (moderate, since $\lambda < \mu^*$) ideological closeness between the bases of parties $B$ and $C$, the amount transferred by $A$ to $C$ in the case of alliance is always less than half of its valuation. This result corroborates the empirical findings of, for example, Kellam (2017) and Golder (2006b), in which political parties are more likely to form electoral coalitions with partners whose ideology is similar to theirs in presidential and legislative elections, respectively.

## 3. Discussion of the auction assumption

The strongest assumption of our model is that leading parties simultaneously make 'take-it-or-leave-it' offers to small ones. As mentioned in the introduction, the literature on coalitions, in general, assumes that negotiations to form alliances occur through bargaining (Bandyopadhyay et al., 2011; Blais and Indridason, 2007; Ferrara and Herron, 2005). Our model, therefore, is novel and different from the standard ones because it focuses on the competition between leading parties for the support of the small ones, instead of negotiation between parties whose bargaining power is not negligible.[23] While it is plausible to think that there may exist certain negotiations between small and leading parties even within a framework like ours, the competition between the latter ones makes the possible bargaining differ from the standard approach.

Under the conditions described by our model, the negotiation through bargaining might occur in the following way: after receiving the first offer from the leading parties—made simultaneously or sequentially—a small party could make counteroffers to each one of them using the offers made by other leading parties to increase its bargaining power. Thus, the small party could make a counteroffer to party $A$, for example, saying something like, 'You are offering me $x$, but party $B$ offered me $y>x$, so I want at least $z>y$.' A similar counteroffer could be made to party $B$ and then another round of negotiation would start. Moreover, given that the offer made by party $A$ is not observed by party $B$, and vice versa, the small party $C$ may have incentives to lie during negotiation. The problem of asymmetric information therefore makes this situation even more complex.

As one can observe, bargaining models such as the one described here are very complex and an object of study in game theory itself (Binmore, 1985). In fact, it is straightforward to see that, as the number of parties increases, the complexity of such bargaining would increase as well. In addition to the intrinsic strategic relationship among the small parties, this type of negotiation requires a longer time to

achieve an agreement than standard models. To see this, observe that once a small party receives all the first offers from leading parties, it must prepare and make counteroffers to every potential ally. Thus, this first round would, in general, take more time than a negotiation between only two potential allies. As we move to the next rounds, the time required to receive and send all the possible counteroffers increases even more.

The main reason that we believe our assumption is reasonable is the lack of available time for such complex and lengthy negotiation. As alliances and coalitions are officially set during political conventions (both national and state), which usually occur close to the election day, there is little time for negotiation among parties.[24] This is particularly true when there are several potential allies. Even for unofficial agreements, made before the conventions, the time available is not enough for bargaining such as described here. The reason is that parties only close the deal when they know who the potential candidates of each leading party are. However, candidacies, in general, are announced during political conventions. Finally, as we have seen, 'take-it-or-leave-it' offers seem to depict the type of bargaining observed when large parties compete for the support of small ones. When the number of small parties is high, their bargaining power is small, a case in which the assumption of 'auction' is even more plausible. In fact, researchers have argued that minor parties may gain more when they form pre-electoral coalitions if they can 'blackmail' major parties by threatening to run independently (Colomer and Negretto, 2005; Kellam, 2017). Our model rules out this possibility, as small parties have no chance of winning by running alone.

## 4. Concluding remarks

The analysis developed here contributes to the understanding of pre-electoral alliances, an area that is still understudied. In particular, we believe that our model provides insights on how those agreements occur in highly fragmented systems, where small parties have negligible bargaining power. Two main factors that may explain why parties form alliances, namely pragmatism and ideology, are analyzed through a sealed-bid first-price auction framework. Although such a framework allows us to perform an accurate analysis and provides a rich intuition about the strategic behavior of parties, it also presents some limitations, which we believe may be overcome in future research and thus generate new insights about the process of alliance formation in politics. Ruling out the assumption of risk neutrality of parties, considering the case of auction with interdependent values—since it is reasonable to assume that parties do not completely know all the sources of benefit and gains of being in office—and allowing coalition among small parties in order to increase their bargaining power are examples of those extensions.

One of the extensions that we believe is particularly promising is to consider other potential sources of alliance loyalty over time—see the empirical evidence presented by Resende (2018). While we focus on the role of the ideological closeness of the parties' bases, there are other possibilities to be explored. One of them may arise if we rule out the assumption of commitment by leading parties, such that they

could promise transfers to small ones but not fulfill them. In this context, we can assume that the small party is uncertain about how trustworthy leading parties are before allying with them. By choosing to ally with a specific party in an election, it can observe whether the leading party fulfills its promise and then update its belief about the type of its ally. Thus, in the next election its decision will be taken based on that information; if, for example, the leading party promised a certain transfer and did not fulfill it, the small party will probably not ally with it again.

The uncertainty about how trustworthy the leading parties are may have an even stronger effect on the alliance loyalty over time if the small parties are risk averse. In this case, there would be a resistance to change a small party's current alliance, owing to the distrust of other potential allies. The idea is that once allied with some leading party, a small one has more information about it than about any other party, which makes the decision of starting a new alliance risky. Finally, if ideology has an important role in the alliance decision, there may be uncertainty about it, such that the real ideology—in contrast to the announced ideology, which can be chosen only to attract voters—of potential allies may be considered unknown. The effects would probably be similar to the previous case and would be affected by the degree of risk aversion of the small parties.

## Acknowledgements

We thank Jefferson Bertolai, Guilherme Stein, seminar participants at FEARP/USP, PPGE/UFRGS and SING14—The 14th European Meeting on Game Theory (Bayreuth, Germany) for their comments and suggestions. All remaining errors are ours.

## Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

## Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by the Brazilian National Council for Scientific and Technological Development (CNPq) (grant number 305838/2017-2).

## ORCID iD

Marcelo de C Griebeler https://orcid.org/0000-0001-7943-4802

## Notes

1. It is important to understand the meaning of 'liberal' in the Brazilian political context. Parties considered to be liberals in Brazil are, in general, 'conservative liberals', such that they are used to combining liberal values and policies with conservative stances, or, more simply, representing the right wing of the liberal movement.
2. There are several different party size measures. When one considers the total number of votes received, the number of participation in elections, or even the number of

candidates elected, PL may be considered a medium party. Nonetheless, we chose to classify it as small owing to its *relative* size, compared with the few large parties that dominate the Brazilian political system.

3. Communist Party of Brazil, Brazilian Communist Party, and Party of the National Mobilization, respectively.
4. In 2006, PL merged with the Party of the National Order Rebuilding (PRONA). The party that emerged from this merger was the Party of the Republic (PR), whose ideological orientation was the same as that of PL, such that it kept its role in the alliance. In addition, in 2005, José Alencar left PL and joined the Brazilian Republican Party (PRB), a political group that he had helped to found in 2003.
5. There are many other important examples of political alliances formed pragmatically throughout history, some going back to ancient societies. The so-called First Triumvirate is perhaps the most famous one. It was an informal political alliance of three prominent men between 59 and 53 BC, during the late Roman Republic: Gaius Julius Caesar, Gnaeus Pompeius Magnus (Pompey the Great), and Marcus Licinius Crassus. While there are several versions of how the alliance came about in the sources, they all agree that Caesar, Pompeius, and Crassus did not share political ideas—in fact, Pompeius and Crassus considered each other enemies. With this alliance, they aimed to overcome the Senate's resistance to some bills that they had sponsored and to have them passed.
6. Throughout the paper, we use the term 'pragmatism' to refer to the office-seeking behavior of political parties. *Besides being widespread in the literature, we chose such an interpretation because* of the idea of the political pragmatism philosophy, in which, 'If it works, we don't really care why.' Once again, this seems to be a good depiction of the Brazilian scenario.
7. The increasing number of political parties in Brazil has been pointed out as one of the main reasons for the decrease in the importance of ideology as a determinant of alliance over time (Resende, 2018). As a result, other potential factors, such as, pragmatism have been increasingly highlighted. In fact, in Brazil, there is the so-called *fundo partidário*, a type of party subsidy paid by the Brazilian government directly to political parties to fund some or all of their political activities—to cover not only campaign costs but routine ones as well—proportional to the number of seats in the Congress. This public funding creates incentives for the presence of a large number of parties. One cannot state, however, that ideology is not particularly salient for a given number of effective parties in all cases, as it may be the key factor that presumably drives party fragmentation in the first place.
8. Conceptually, an electoral alliance may take the form of an electoral agreement with two (bipartisan) or more parties (electoral bloc). In this paper, we use the concept in its former form, that is, a bipartisan electoral agreement, as the maximum number of parties allowed in an alliance in our model is two. As most of the literature on bloc alliances adopt the term 'coalition', we believe that by using 'alliance' we avoid misinterpretation.
9. The list of other countries whose system is coalitional presidentialism includes Armenia, Benin, Chile, Ecuador, Kenya, Malawi, Russia, and Ukraine (Chaisty et al., 2018).
10. Since the re-democratization, in 1986, the number of political parties has monotonically increased in Brazil—the only exception was the 1998 election. However, historically, ENP/$N$ has been around 0.5. In the last two presidential elections (2014 and 2018), for example, there were 28 and 30 parties, while the ENP was 13.4 and 16.4,

respectively. According to Freire (2018), Brazil has the second most fragmented system in the world—the most fragmented system being that of Papua New Guinea.

11. Despite the importance of the system of open-list proportional representation, it is important to note that our model does not analyze it. Instead, we focus on the presidential system, which allows us to explore the two factors previously discussed. To take into account the characteristics of a proportional system, a model would have to include simultaneous elections at different levels (e.g. state and federal) for different offices (president, governor, congressman, etc.), which is beyond of the scope of this study. Further research on this topic would benefit from incorporating the proportional part of the system. For an empirical analysis of the institutional incentives that induce political parties to form pre-electoral alliances under mixed electoral rules, see, for example, Ferrara and Herron (2005).
12. It is worth noticing that the electoral pacts between PRI and PVEM are similar to the Brazilian ones, cited previously, given that the two parties do not share an ideology and that their bases of support come from different sectors.
13. In the baseline model, we assume that $\mu$ is the same for both parties; that is, the share of party $C$ voters which follows it in an alliance does not depend on the specific ally. In the next section, we relax this assumption by allowing ideology to affect the share of voters attracted by each alliance.
14. Observe that in our simple environment, the fact that disloyal voters end up abstaining or casting an invalid or blank vote does not affect the final outcome. However, this may not be true for all electoral systems.
15. Without loss of generality, we assume that in the case of a tie in the election, $A$ (or its alliance) wins.
16. This assumption is supported by the literature, which shows that parties often invest significant resources to measure the size of the electorate and other benefits associated with possible alliances (Kaminski, 2001). This may be done through surveys performed by private pooling companies, which ask voters whether they support a given alliance (Kaminski, 2002), or by using alliances formed at the regional level as experiments to evaluate their potential performance in a national election (Downs, 1998).
17. Observe that, given that by forming an alliance the probability of winning is greater than or equal to the probability of running individually, we are implicitly assuming that alliances are super-additive, that is, parties' electoral performance is better jointly than individually. See Kaminski (2001) for more on the categorization of alliances as super-additive, additive, or sub-additive.
18. Given that the decision of alliance is made by party leaders (or party elites), this assumption implies that politicians are purely office-seekers. In fact, the anecdotal evidence about the Brazilian political scenario, our main motivation, supports it: for example, over the last 20 years parties with different ideologies—ideologies announced in their names or statutes—have adopted the same economic and social policies.
19. However, this is not the only reason why a government might decide to offer high-level positions to politicians of a different party. This strategy can also be used to form a coalition and increase the strength of the allied base, which in turn increases the relative power of the executive government (Ames, 2002).
20. It may be hard to envision how such commitments would be possible in a one-shot election, so $C$ might be suspicious of campaign promises. In fact, researchers such as Kellam (2017) assume that presidents are not required to honor any pre-electoral agreement. However, leading parties might have an incentive to carry through on their promises if they intend to compete in more than one election and if they care about

their reputation for telling the truth (Mitchell, 1999). In addition, in coalitional presidentialism, the implementation of the government's agenda relies on legislative support, which can be achieved by honoring the promises made to its allies during a campaign (Borges and Peron, 2018). Nevertheless, although part of the literature (Laver and Schofield, 1998; Strom et al., 1994) argues that questions of credibility and commitment are rarely a problem in practice, even when discussions of electoral coordination do not involve nomination agreements, we are aware that such an assumption may be strong in some contexts. For example, the fact that those problems are not observed may be due to a selection problem: parties confronted by sufficiently challenging commitment problems do not form alliances.

21. One can conjecture that if we allow the voters of $A$ and $B$ to be disloyal, the result of Proposition 1 will change. To see this, suppose that the share of loyal voters of both parties is also a random variable. Thus, it is possible that an alliance with $C$ makes the voters of $A$ more unsatisfied than those of $B$—because of ideological reasons, for example. If the probability of this event is high enough, it is possible that the effect overcomes the ex-ante advantage of $A$. In fact, $A$ may lose so many voters that the alliance with $C$ might not be a good choice. The relation between voters' loyalty to parties and electoral alliances seems to be a promising field for further research.
22. Observe that we do not model ideology. Instead, it is assumed to be exogenous. Given that our setting is static, we can interpret $\lambda$ as a fixed (constant) measure of how much voters' beliefs, especially those that form the basis of their economic and political ideals, are close.
23. Although political negotiations are often conducted behind closed doors, we can find some anecdotal evidence of the prevalence of such a competition in Brazil through the press. Consider, for instance, the following news headlines: 'Lula and Alckmin dispute the support of PMDB in the second round' ('Lula e Alckmin já disputam o apoio do PMDB no segundo turno', in Portuguese) (Matais and Neves, 2006), regarding the 2006 presidential elections, and 'Disputed by Serra and Dilma, PV will decide support in convention' ('Disputado por Serra e Dilma, PV decidirá apoio em convenção', in Portuguese) (BBC, 2010), regarding the 2010 elections.
24. To have an idea of the little time available to form pre-electoral coalitions, consider the 2018 Brazilian elections: political conventions should occur between July 20th and August 8th, therefore there are only 20 days for official negotiation.

## References

Ames B (2002) *The Deadlock of Democracy in Brazil*. Ann Arbor, MI: University of Michigan Press.

Bandyopadhyay S, Chatterjee K and Sjöström T (2011) Pre-electoral coalitions and post-election bargaining. *Quarterly Journal of Political Science* 6(1): 1–53.

Batista M (2018) Who gets what and how does it matter? Importance-weighted portfolio allocation and coalition support in Brazil. *Journal of Politics in Latin America* 10(3): 99–134.

BBC (2010) Disputado por Serra e Dilma, PV decidirá apoio em convenção. Available at: https://www.bbc.com/portuguese/noticias/2010/10/101004_eleicoes_partidoverde_rp (accessed 20 January 2018).

Binmore KG (1985) Bargaining and coalitions. In: Roth AE (ed.) *Game-theoretic Models of Bargaining*. Cambridge: Cambridge University Press.

Blais A and Indridason IH (2007) Making candidates count: The logic of electoral alliances in two-round legislative elections. *The Journal of Politics* 69(1): 193–205.

Borges A and Peron I (2018) Repensando o presidencialismo de coalizão: Coligações eleitorais e formação de governos no Brasil em perspectiva comparada. *Anais do 42º Encontro Anual da ANPOCS*, Caxambut, Brazil, 22–26 October 2018. Caxambu, Brazil: Anais da ANPOCS.

Borges T, Carreirão Y and Nascimento F (2017) Os partidos políticos brasileiros e a (in) consistência das coligações para deputado federal em 2014. In: Krause S, Machado C and Miguel L (eds.) *Coligações e disputas eleitorais na Nova República: Aportes teórico-metodológicos, tendências e estudos de caso*. Rio de Janeiro: Konrad Adenauer Stiftung.

Chaisty P, Cheeseman N and Power TJ (2018) *Coalitional Presidentialism in Comparative Perspective: Minority Presidents in Multiparty Systems*. Oxford: Oxford University Press.

Colomer JM and Negretto GL (2005) Can presidentialism work like parliamentarism? *Government and Opposition* 40(1): 60–89.

De Lima Júnior OB (1983) *Os partidos políticos brasileiros: A experiência federal e regional, 1945–1964*. Rio de Janeiro: Graal.

De Miranda GL (2013) Coligações eleitorais: Tendências e racionalidades nas eleições federais e majoritárias estaduais (1990–2010). *Revista de Sociologia e Política* 21(47): 69–90.

De Oliveira IRx (1973) Notas sobre o comportamento das coligações eleitorais no Brasil (1950–1962). *Dados* (10): 166–183.

Dos Santos WG (1987) *Crise e castigo: Partidos e generais na política brasileira*. São Paulo: Vértice, 134.

Downs WM (1998) *Coalition Government, Subnational Style: Multiparty Politics in Europe's Regional Parliaments*. Columbus, OH: Ohio State University Press.

Ferrara F and Herron ES (2005) Going it alone? Strategic entry under mixed electoral rules. *American Journal of Political Science* 49(1): 16–31.

Fréchette GR, Kagel JH and Morelli M (2005) Gamson's law versus non-cooperative bargaining theory. *Games and Economic Behavior* 51(2): 365–390.

Freire VT (2018) Fragmentação de partidos, recorde, é aberração mundial. *Folha de São Paulo*. Available at: https://www1.folha.uol.com.br/poder/2018/10/fragmentacao-de-partidos-recorde-e-aberracao-mundial.shtml (accessed 20 January 2018).

Golder SN (2005) Pre-electoral coalitions in comparative perspective: A test of existing hypotheses. *Electoral Studies* 24(4): 643–663.

Golder SN (2006a) *The Logic of Pre-electoral Coalition Formation*. Columbus, OH: Ohio State University Press.

Golder SN (2006b) Pre-electoral coalition formation in parliamentary democracies. *British Journal of Political Science* 36(2): 193–212.

Golosov GV (2010) The effective number of parties: A new approach. *Party Politics* 16(2): 171–192.

Kaminski MM (2001) Coalitional stability of multi-party systems: Evidence from Poland. *American Journal of Political Science* 45(2): 294–312.

Kaminski MM (2002) Do parties benefit from electoral manipulation? Electoral laws and heresthetics in Poland, 1989–93. *Journal of Theoretical Politics* 14(3): 325–358.

Kellam M (2017) Why pre-electoral coalitions in presidential systems? *British Journal of Political Science* 47(2): 391–411.

Krause S and Godoi P (2010) Coligações eleitorais para os executivos estaduais (1986–2006): Padrões e tendências. In: Krause S, Dantas H and Miguel LF (eds.) *Coligações*

*partidárias na nova democracia brasileira: Perfis e tendências*. Rio de Janeiro: Fundação Konrad Adenauer/UNESP.

Krishna V (2009) *Auction Theory*. 2nd ed. Burlington, MA: Academic Press.

Laakso M and Taagepera R (1979) "Effective" number of parties: A measure with application to West Europe. *Comparative Political Studies* 12(1): 3–27.

Lavareda A (1991) *A democracia nas urnas: O processo partidário eleitoral brasileiro*. Rio de Janeiro: IUPERJ/Rio Fundo.

Laver M and Schofield N (1998) *Multiparty Government: The Politics of Coalition in Europe*. Ann Arbor, MI: University of Michigan Press.

Machado A (2009) Minimum winning electoral coalitions under presidentialism: Reality or fiction? The case of Brazil. *Latin American Politics and Society* 51(3): 87–110.

Machado A (2012) *Alianças eleitorais: Casamento com prazo de validade: O caso das coligações brasileiras*. Rio de Janeiro: Elsevier.

Matais A and Neves F (2006) Lula e alckmin já disputam o apoio do pmdb no segundo turno. *Folha de São Paulo*. Available at: https://www1.folha.uol.com.br/folha/brasil/ult96u84551.shtml (accessed 20 January 2018).

Melo CR (2015) The 2014 elections and the Brazilian party system. *Brazilian Political Science Review* 9(1): 93–114.

Miguel LF and Machado C (2007) Um equilíbrio delicado: A dinâmica das coligações do PT em eleições municipais (2000 e 2004). *Dados-Revista de Ciências Sociais* 50(4): 757–793.

Mitchell P (1999) Government formation: A tale of two coalitions. In: Marsh M and Mitchell P (eds.) *How Ireland Voted 1997*. Oxford: Westview Press, 243–263.

Resende RC (2018) *Um modelo teórico para o estudo de alianças eleitorais*. PhD Thesis, Universidade Federal do Rio Grande do Sul, Brazil.

Resende RC and Epitácio SdS (2017) 'Fidelidade aliancista': As alianças eleitorais nas eleições majoritárias estaduais e o caso do PSB. *Brasiliana—Journal for Brazilian Studies* 6(1): 250–279.

Riker WH (1962) *The Theory of Political Coalitions*. New Haven, CT: Yale University Press.

Schmitt R (1999) *Coligações eleitorais e sistema partidário no Brasil*. PhD Thesis, Instituto Universitário de Pesquisas do Rio de Janeiro, Brazil.

Soares GAD (1964) Alianças e coligações eleitorais: Notas para uma teoria. *Revista Brasileira de Estudos Políticos* 17: 95–124.

Spoon JJ and Gómez AP (2017) Unusual bedfellows? PRI-PVEM electoral alliances in Mexican legislative elections. *Journal of Politics in Latin America* 9(2): 63–92.

Spoon JJ and West KJ (2015) Alone or together? How institutions affect party entry in presidential elections in Europe and South America. *Party Politics* 21(3): 393–403.

Strom K, Budge I and Laver MJ (1994) Constraints on cabinet formation in parliamentary democracies. *American Journal of Political Science* 38(2): 303–335.

van de Wardt M (2017) Explaining the effective number of parties: Beyond the standard model. *Electoral Studies* 45: 44–54.