TOPICS IN MICROECONOMICS

This text in microeconomics focuses on the strategic analysis of markets under imperfect competition, incomplete information, and incentives.

Part I of the book covers imperfect competition, from monopoly and regulation to the strategic analysis of oligopolistic markets. Part II explains the analytics of risk, stochastic dominance, and risk aversion, supplemented with a variety of applications from different areas in economics. Part III focuses on markets and incentives under incomplete information, including a comprehensive introduction to the theory of auctions, which plays an important role in modern economics.

Each chapter introduces the core issues in an accessible yet rigorous fashion, and then investigates specialized themes. An attempt is made to develop a coherent story, with self-contained explanations and proofs. The only prerequisites are a basic knowledge of calculus and probability, and familiarity with intermediate undergraduate microeconomics.

The text can be used as a textbook in courses on microeconomics, theoretical industrial organization, and information and incentives for senior undergraduate or first-year graduate students.

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INDUSTRIAL ORGANIZATION, AUCTIONS, AND INCENTIVES

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Men prize the thing ungained more than it is.
William Shakespeare

8.1 Introduction

Suppose you inherit a unique and valuable good – say a painting by van Gogh or a copy of the 1776 edition of Adam Smith's *Wealth of Nations*. For some reason you are in desperate need for money, and decide to sell. You have a clear idea of your valuation. Of course, you hope to earn more than that. You wonder: isn't there some way to reach the highest possible price? How should you go about it?

If you knew the potential buyers and their valuations of the object for sale, your pricing problem would have a simple solution. You would only need to call a nonnegotiable price equal to the highest valuation, and wait for the right customer to come and claim the item. It's as simple as that.[1]

8.1.1 Information Problem

The trouble is that you, the seller, usually have only incomplete information about buyers' valuations. Therefore, you have to figure out a pricing scheme that performs well even under incomplete information. Your problem begins to be interesting.

8.1.2 Basic Assumptions

Suppose there are $n > 1$ potential buyers, each of whom knows the object for sale well enough to decide how much he is willing to pay. Denote buyers' valuations by $V := (V_1, \ldots, V_n)$. Since you, the seller, do not know V, you must think of it as a random variable, distributed by some joint probability distribution function (CDF) $\mathcal{F} : [\underline{v}_1, \bar{v}_1] \times \cdots \times [\underline{v}_n, \bar{v}_n] \to [0, 1]$, $\mathcal{F}(v_1, \ldots, v_n) := \Pr(V_1 \leq v_1, \ldots, V_n \leq v_n)$. Similarly, potential buyers know their own valuation but not that of others. Therefore, buyers also view valuations as a random variable, except their own.

[1] Of course, it is not really easy to make something nonnegotiable.

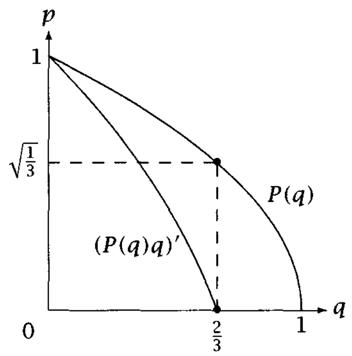

Figure 8.1. Optimal Take-It-Or-Leave-It Price When There Are Two Potential Buyers.

8.1.3 Cournot-Monopoly Approach

Of course, you could stick to the take-it-or-leave-it pricing rule, even as you are subject to incomplete information about buyers' valuations. You would then call a nonnegotiable price p at or above your *own valuation* r, and hope that some customer has the right valuation and is willing to buy. Your pricing problem would then be reduced to a variation of the standard Cournot monopoly problem. The only unusual part would be that the trade-off between price and quantity is replaced by one between price and the probability of sale.

Alluding to the usual characterization of Cournot monopoly, you can even draw a "demand curve," with the nonnegotiable price p on the "price axis" and the probability that at least one bidder's valuation exceeds the listed price p

$$\pi(p) = 1 - G(p), \quad G(p) := \mathcal{F}(p, \ldots, p), \tag{8.1}$$

on the "quantity axis" (as in Figure 8.1). You would then pick that price p that maximizes the expected value of your gain from trade, $\pi(p)(p - r)$.

Equivalently, you can state the decision problem in quantity coordinates, as is customary in the analysis of Cournot monopoly, where "quantity" is represented by the probability of sale, $q := 1 - G(p)$:

$$\max_q \, (P(q) - r)\, q, \qquad P(q) := G^{-1}(1 - q). \tag{8.2}$$

Computing the first-order condition, the optimal price p^* can then be characterized in the familiar form as a relationship between *marginal revenue* (MR) and *marginal cost* (MC):[2]

$$\mathrm{MR} := p - \frac{1 - G(p)}{G'(p)} = r =: \mathrm{MC}. \tag{8.3}$$

Example 8.1 Suppose valuations V_i are independent and uniformly distributed over the interval $[0, 1]$, and the seller's own valuation is $r = 0$. Then, for all $p \in [0, 1]$,

[2] As always, this condition applies only if the maximization problem is well behaved. Well behavedness requires that revenue be continuous and marginal revenue strictly monotone increasing in p.

$G(p) = F(p)^n = p^n$. Hence, the optimal take-it-or-leave-it price is

$$p^*(n) = \sqrt[n]{\frac{1}{n+1}}. \tag{8.4}$$

The resulting expected price paid (price asked times probability of success) is

$$\bar{p}(n) := p^*(n)(1 - G(p^*(n))) = p^*(n)\frac{n}{n+1}. \tag{8.5}$$

Both $p^*(n)$ and $\bar{p}(n)$ are strictly monotone increasing in n, starting from $p^*(1) = 1/2$, $\bar{p}(1) = 1/4$, and approaching 1 as $n \to \infty$.[3]

Take-it-or-leave-it pricing is one way to go. But generally you can do better by setting up an auction. This brings us to the analysis of auctions, and the design of optimal auction rules. Interestingly, the particular Cournot monopoly price $p^*(1)$ that holds for $n = 1$, will continue to play a role as an optimal minimum price in the optimal auction setting, as you will learn in the section on optimal auctions.

8.1.4 Auctions – What, Where, and Why

An auction is a bidding mechanism, described by a set of auction rules that specify how the winner is determined and how much he has to pay. In addition, auction rules may restrict participation and feasible bids and impose certain rules of behavior.

Auctions are widely used in many transactions – not just in the sale of art and wine. Every week, the U.S. Treasury auctions-off billions of dollars of bills, notes, and bonds. Governments and private corporations solicit delivery-price offers on products ranging from office supplies to tires and construction jobs. The Department of the Interior auctions off the rights to drill oil and other natural resources on federally owned properties. And private firms auction off products ranging from fresh flowers, fish, and tobacco to diamonds and real estate. Altogether, auctions account for an enormous volume of transactions.

Essentially, auctions are used for three reasons:

- speed of sale,
- to reveal information about buyers' valuations,
- to prevent dishonest dealing between the seller's agent and the buyer.

The agency role of auctions is particularly important if government agencies are involved in buying or selling. As a particularly extreme example, think of the recent privatization programs in Eastern Europe. Clearly, if the agencies involved, such as the *Treuhandanstalt* that was in charge of privatizing the state-run corporations of formerly communist East Germany, had been free to negotiate the terms of sale, the lucky winner probably would have been the one who made the largest bribe or political contribution. However, if assets are put up for auction, cheating the taxpayer is more difficult and costly, and hence less likely to succeed.

[3] For a simple proof of monotonicity, work with $\ln \bar{p}$, which is obviously a monotone transformation of $\bar{p}$.

Table 8.1. *The most popular auctions*

Open	Closed
Ascending-price (English)	Second-price (Vickrey)
Descending-price (Dutch)	First-price

8.1.5 Popular Auctions

There are many different auction rules. Actually, the word itself is something of a misnomer. *Auctio* means increase but not all auctions involve calling out higher and higher bids.

One distinguishes between *oral* and *written* auctions. In oral auctions, bidders hear each other's bids and can make counteroffers; each bidder knows his rivals. In closed auctions, bidders submit their bids simultaneously without revealing them to others; often bidders do not even know how many rival bidders participate.

The best known and most frequently used auction (Table 8.1) is the ascending price or English auction, followed by the first-price closed-seal bid or Dutch auction, and the second-price closed-seal bid (also known as Vickrey auction).[4]

In the ascending-price or *English* auction, the auctioneer seeks increasing bids until all but the highest bidder(s) are eliminated. If the last bid is at or above the minimum price, the item is awarded or *knocked down* to the remaining bidder(s). If a tie bid occurs, the item is awarded for example by a chance rule. In one variation, used in Japan, the price is posted on a screen and raised continuously. Any bidder who wants to be active at the current price pushes a button. Once the button is released, the bidder has withdrawn and is not permitted to reenter the bidding.

Instead of letting the price rise, the descending-price or *Dutch* auction follows a descending pattern. The auctioneer begins by asking a certain price, and gradually lowers it until some bidder(s) shout "Mine" to claim the item. Frequently, the auctioneer uses a mechanical device called the *Dutch clock*. This clock is started, and the asking price ticks down until someone calls out. The clock then stops and the buyer pays the indicated price. Again, provisions are made to deal with tie bids.[5]

Finally, in a written auction bidders are invited to submit a sealed bid, with the understanding that the item is awarded to the highest bidder. Under the first-price rule the winner actually pays as much as his own bid, whereas under the second-price rule the price is equal to the second highest bid.

[4] Many variations of these basic auctions are reviewed in Cassady (1967).

[5] An interesting Dutch auction in disguise is *time discounting*. In many cities in the U.S. discounters use this method to sell clothes. Each item is sold at the price on the tag minus a discount that depends on how many weeks the item was on the shelf. As time passes, the price goes down by (say) 10% per week, until the listed bottom price is reached.

This terminology is not always used consistently in the academic, and in the financial literature. For example, in the financial community a multiple-unit, single-price auction is termed a Dutch auction, and a multiple-unit, sealed bid auction is termed an English auction (except by the English, who call it an American auction). In the academic literature, the labels English and Dutch would be exactly reversed. In order to avoid confusion, keep in mind that here we adhere to the academic usage of these terms, and use English as equivalent to ascending and Dutch as equivalent to descending price.

8.1.6 Early History of Auctions

Probably the earliest report on auctions is found in Herodotus in his account of the bidding for men and wives in Babylon around 500 B.C. These auctions were unique, since bidding sometimes started at a negative price.[6] In ancient Rome, auctions were used in commercial trade, to liquidate property, and to sell plundered war booty. A most notable auction was held in 193 A.D. when the Praetorian Guard put the whole empire up for auction. After killing the previous emperor, the guards announced that they would appoint the highest bidder as the next emperor. Didius Julianus outbid his competitors, but after two months was beheaded by Septimius Severus, who seized power – a terminal case of the winner's curse?

8.2 The Basics of Private-Value Auctions

We begin with the most thoroughly researched auction model, the *symmetric independent private values* (SIPV) model with risk-neutral agents. In that model

- a single indivisible object is put up for sale to one of several bidders (*single-unit auction*);
- each bidder knows his valuation – no one else does (*private values*);
- all bidders are indistinguishable (*symmetry*);
- unknown valuations are independent and identically distributed (iid) and continuous random variables (*independence, symmetry, continuity*);
- bidders are *risk-neutral*, and so is the seller (unless stated otherwise).

We will model bidders' behavior as a noncooperative game under incomplete information. One of our goals will be to rank the four common auctions. To what extent does institutional detail matter? Can the seller get a higher average price by an oral or by a written auction? Is competition between bidders fiercer if the winner pays a price equal to the second highest rather than the highest bid? And which auction, if any, is strategically easier to handle?

8.2.1 Some Basic Results on Dutch and English Auctions

The comparison between the four standard auction rules is considerably facilitated by the fact that the Dutch auction is equivalent to the first-price and the English

[6] See Shubik (1983).

to the second-price closed auction. Therefore, the comparison of the four standard auctions can be reduced to comparing the Dutch and the English auction. Later you will learn the far more surprising result that even these two auctions are payoff-equivalent.

The strategic equivalence between the Dutch auction and the first-price closed auction is immediately obvious. In either case, a bidder has to decide how much he should bid or at what price he should shout "Mine" to claim the item. Therefore, the strategy space is the same, and so are the payoff functions and hence the equilibrium outcomes.

Similarly, the English auction and the second-price closed auction are equivalent, but for different reasons and only as long as we stick to the private-values framework. Unlike in a sealed-bid auction, in an English auction bidders can respond to rivals' bids. Therefore, the two auction games are not strategically equivalent. However, in both cases bidders have the obvious (weakly) dominant strategy to bid an amount or set a limit equal to their own true valuation. Therefore, the equilibrium outcomes are the same as long as bidders' valuations are not affected by observing rivals' bidding behavior, which always holds true in the private-values framework.[7]

The essential property of the English auction and the second-price closed auction is that the price paid by the winner is exclusively determined by rivals' bids. Bidders are thus put in the position of price takers who should accept any price up to their own valuation. The *truth-revealing* strategy where one always bids one's own valuation is a (weakly) dominant strategy. The immediate implication is that both auction forms have the same equilibrium outcome, provided bidders never play a weakly dominated strategy.

8.2.2 Revenue Equivalence

We have established the payoff equivalence of Dutch and first-price and of English and second-price auctions. Remarkably, payoff equivalence is a much more general feature of auction games. Indeed, *all* auctions that award the item to the highest bidder and lead to the same bidder participation are payoff-equivalent. The four standard auctions are a case in point. We now turn to the proof of this surprising result and then explore modifications of the SIPV framework.

We mention that the older literature conjectured that Dutch auctions lead to a higher average price than English auctions because, as Cassady (1967, p. 260) put it, in the Dutch auction "... each potential buyer tends to bid his highest demand price, whereas a bidder in the English system need advance a rival's offer by only one increment." Both the reasoning and the conclusion are wrong.

Before we go into the general solution of SIPV auction games, in the following subsection we elaborate on an extensive example and solve several standard auction

[7] Obviously, if rivals' bids signal something about the underlying common value of the item, each bidder's estimated value of the item is updated in the course of an English auction. No such updating can occur under a sealed-bid auction, where bidders place their bids before observing rivals' behavior. Therefore, if the item has a *common* value component, the English auction and the second-price sealed bid do not have the same equilibrium outcome.

games for the case of uniformly distributed valuations. This serves as motivation for our general results and as a playground to exercise the standard solution procedure that is typically employed in the literature.

The standard procedure to solve a particular auction, as in any other game, is to first solve for equilibrium strategies and then compute equilibrium payoffs. This procedure can, of course, be applied to arbitrary probability distribution functions, not just to uniform distributions. But when we come to the general solution of SIPV auction games, we will switch – with good reason – to an alternative solution procedure. Since you can learn from both, it is advisable neither to jump immediately to the general solution in Section 8.2.5 nor to skip the latter.

8.2.3 Solution of Some Auction Games – Assuming Uniformly Distributed Valuations

Example 8.2 Suppose all bidders' valuations are uniformly distributed on the support $[0, 1]$, as in Example 8.1, and the seller's own valuation and minimum bid are equal to zero. Then the unique equilibrium bid functions $b^* : [0, 1] \to \mathbb{R}_+$ are[8]

$$b^*(v) = v \qquad \text{(English auction)}, \tag{8.6}$$

$$b^*(v) = \left(1 - \frac{1}{n}\right) v \qquad \text{(Dutch auction)}. \tag{8.7}$$

In both auctions, the expected price is

$$\bar{p} = \frac{n-1}{n+1}, \tag{8.8}$$

and the equilibrium payoffs are

$$U^*(v) = \frac{v^n}{n} \qquad \text{(bidders' expected gain)}, \tag{8.9}$$

$$\Pi = \bar{p} = \frac{n-1}{n+1} \qquad \text{(seller's expected gain)}. \tag{8.10}$$

This implies payoff equivalence. Moreover, more bidders mean fiercer competition, to the seller's advantage and to the bidders' disadvantage.

We now explain in detail how these results come about. The proof uses two preliminary assumptions that will then be confirmed to hold: the symmetry of equilibrium and the strict monotonicity of equilibrium strategies.

Dutch (First-Price) Auction

Suppose bidder i bids the amount b, and all rival bidders bid according to the strictly monotone increasing equilibrium strategy $b^*(v)$. Then bidder i wins if and only if all rivals' valuations are below $b^{*-1}(b)$. Since the V's are continuous random variables and $b^*(v)$ is strictly monotone increasing, ties have zero probability. Therefore, the

[8] Uniqueness of the English auction assumes elimination of weakly dominated strategies. More on this below.

probability of winning when bidding the amount b against rivals who play the strategy $b^*(v)$ is

$$\begin{aligned}\rho(b) &= \Pr\{b^*(V_j) < b,\ \forall j \neq i\} \\ &= \Pr\{V_j < \sigma(b),\ \forall j \neq i\}, \qquad \sigma := b^{*-1} \\ &= F(\sigma(b))^{n-1}. \end{aligned} \tag{8.11}$$

Here $\sigma(b)$ is the inverse of $b^*(v)$, which indicates the valuation that leads to bidding the amount b when strategy $b^*(v)$ is played.

In equilibrium, the bid b must be a best response to rivals' bidding strategies. Therefore, b must be a maximizer of the expected utility

$$U(b, v) := \rho(b)(v - b), \tag{8.12}$$

leading to the first-order condition

$$\frac{\partial}{\partial b} U(b, v) = \rho'(b)(v - b) - \rho(b) = 0. \tag{8.13}$$

Moreover, b itself must be prescribed by the equilibrium strategy $b^*(v)$ or, equivalently, one must have

$$v = \sigma(b). \tag{8.14}$$

And a bidder with valuation 0 should bid zero, or equivalently

$$\sigma(0) = 0. \tag{8.15}$$

Using (8.14), one can rewrite (8.13) in the form

$$(n-1) f(\sigma(b))\,(\sigma(b) - b)\,\sigma'(b) - F(\sigma(b)) = 0. \tag{8.16}$$

This simplifies, due to $F(\sigma(b)) = \sigma(b)$ (uniform distribution), to

$$(n-1)(\sigma(b) - b)\sigma'(b) - \sigma(b) = 0. \tag{8.17}$$

Given the initial condition $\sigma(0) = 0$, this differential equation has the obvious solution $\sigma(b) = [n/(n-1)]b$.[9] Finally, solve this for b and you have the asserted equilibrium strategy[10]

$$b^*(v) = \left(1 - \frac{1}{n}\right) v. \tag{8.18}$$

Notice that the margin of profit that each player allows himself in his bid decreases as the number of bidders increases.

Based on this result, you can now determine the expected price $\bar{p}(n)$ by the following consideration: In a Dutch auction, the random equilibrium price, P_D, is equal to the highest bid, which can be written as $b^*(V_{(n)})$, where $V_{(n)}$ denotes the

[9] Why is it obvious? Suppose the solution is linear; then you will easily find the solution, which also proves that there is a linear solution.

[10] This raises the question whether the equilibrium is unique. The proof is straightforward for $n = 2$. In this case, one obtains from (8.17) $\sigma\, d\sigma = \sigma\, db + b\, d\sigma = d(\sigma b)$. Therefore, $\sigma b = \frac{1}{2}\sigma^2 + \sigma(0)$. Since $b(0) = 0$, one has $\sigma(0) = 0$ and thus $\sigma(b) = 2b$. Hence, the unique equilibrium strategy is $b^*(v) = \frac{1}{2}v$. In order to generalize the proof to $n > 2$, apply a transformation of variables from (σ, b) to (z, b), where $z := \sigma/b$. Then one can separate variables and uniquely solve the differential equation by integration.

highest order statistic of the entire sample of n valuations. Therefore, by a known result on order statistics,[11]

$$
\begin{aligned}
\bar{p}(n) &= E\left[b^*\left(V_{(n)}\right)\right] \\
&= \frac{n-1}{n} E\left[V_{(n)}\right] \qquad \left(\text{since } b^*(v) = \tfrac{n-1}{n} v\right) \\
&= \frac{n-1}{n+1},
\end{aligned} \tag{8.19}
$$

as asserted.

English (Second-Price) Auction

Recall that in an English auction truthful bidding, $b(v) \equiv v$, is a weakly dominant strategy. Therefore, if one eliminates weakly dominated strategies, the bidder with the highest valuation wins and pays a price equal to the second highest valuation. The random equilibrium price P_E is equal to the order statistic $V_{(n-1)}$ of the given sample of n valuations. And the expected price is

$$\bar{p}(n) = E\left[V_{(n-1)}\right] = \frac{n-1}{n+1}. \tag{8.20}$$

Notice, however, that the English auction is plagued by a multiplicity problem. For example, the "wolf–sheep" strategy combination

$$b_1(v) = 1, \qquad b_i(v) = 0, \quad i = 2, \ldots, n,$$

where bidder 1 bids very aggressively and bidders 2 to n hold back, is also an equilibrium, even though it prescribes rather pathological behavior. These equilibria are overlooked if one eliminates weakly dominated strategies. (This is why game theorists view the elimination of *weakly* dominated strategies as problematic.)

Can one eliminate these pathological equilibria with a plausible equilibrium refinement? Yes, one can. These equilibria fail the test of *trembling-hand perfection* (see Fudenberg and Tirole (1991)).

Comparison

Since the two expected prices are the same, this example illustrates the general principle of revenue equivalence. (It is left to the reader to confirm the stronger payoff equivalence, asserted in (8.9) and (8.10).[12]) Notice, however, that random prices differ in their risk characteristics. Indeed, the random equilibrium price in the Dutch auction, P_D, second-order stochastically dominates that of the English auction, P_E. Therefore, all *risk-averse* sellers unanimously prefer the Dutch to the English auction.

[11] Let $V_{(r)}$ be the rth order statistic of a given sample of n independent and identically distributed random variables V_i with distribution function $F(v) =, v$ on the support $[0, 1]$. Then $E[V_{(r)}] = r/(n+1)$ and $\mathrm{Var}(V_{(r)}) = r(n-r+1)/(n+1)^2(n+2)$. See rules (R18), (R19) in Appendix D, Section D.4.

[12] Note that in an English auction $U^*(v) = F(v)^{N-1}(v - E[V_{(n-1)} \mid V_{(n)} = v])$; use (D.42), from Appendix B, to compute the conditional expected value.

To illustrate this, recall that one random variable second-order stochastically dominates another if their expected values are the same and their probability distribution functions (cdf's) are single-crossing. This result is often referred to as the *mean-preserving spread* theorem. A mean-preserving spread of a given distribution shifts probability mass towards both tails of the distribution. It is plausible that this induces *more* risk (for a proof see Section 4.4.2). Since the seller prefers higher prices, the dominated random variable is the one that has more probability mass on the tails of the distribution.

The cdf's of $P_D := b^*(V_{(n)})$ and of $P_E := V_{(n-1)}$ are

$$F_D(x) = \Pr\left\{b^*\left(V_{(n)}\right) < x\right\} = \min\left\{\left(\frac{n}{n-1}\right)^n x^n, 1\right\}, \tag{8.21}$$

$$F_E(x) = \Pr\left\{V_{(n-1)} < x\right\} = \min\{x^n + n(x^{n-1} - x^n), 1\}. \tag{8.22}$$

Consider the difference

$$\phi(x) := F_D(x) - F_E(x) = \left(\left(\frac{n}{n-1}\right)^n + (n-1)\right)x^n - nx^{n-1}. \tag{8.23}$$

Evidently, $\phi(x)$ is an nth-order polynomial with only one change in the sign of its coefficients. By Descartes's rule, $\phi(x)$ has at most one real positive root. Also, $\phi(x)$ must have at least one real root in $(0, 1)$ because the expected values of P_D and P_E are the same. Hence, $\phi(x)$ has exactly one real root in $(0, 1)$. This confirms single crossing.

Finally, observe that P_E has more probability mass on both tails of the distribution than P_D because

$$1 - F_E(x) > 1 - F_E(1) = 0 = 1 - F_D(x) \qquad \forall x \in \left[\frac{n-1}{n}, 1\right],$$

as illustrated in Figure 8.2. Therefore, P_D second-order stochastically dominates P_E.

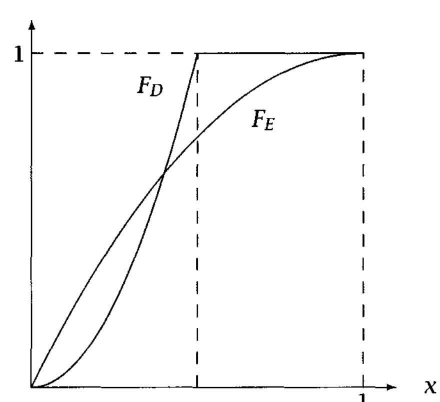

Figure 8.2. Probability distributions of P_D, P_E for $n = 2$.

*Third- and Higher-Price Auctions**

Consider a generalization of the second-price auction: the *third-price* auction, where the highest bidder wins but pays only as much as the third highest bid, and, more generally the *k th-price* auction, where the winner pays the kth highest bid.[13] It has the following solution:[14]

$$b_k^*(v) = \frac{n-1}{n-k+1}v, \qquad \text{for} \quad n > 2, \quad k = 1, \ldots, n. \tag{8.24}$$

Third- and higher-price auctions have four striking properties: 1) bids are higher than the own valuation; 2) equilibrium bids increase when one moves to a higher-price auction; 3) equilibrium bids diminish as the number of bidders is increased (since $(n-1)/(n-k+1) > n/(n-k+2)$); and 4) the variance of the random price P_k,

$$\mathrm{Var}(P_k) = \mathrm{Var}\big(b_k^*(V_{(n-k+1)})\big) = \frac{k(n-1)}{(n-k+1)(n+1)^2(n+2)^2}, \tag{8.25}$$

increases in k, and more generally, P_k second-order stochastically dominates P_{k+1} for all $k = 1, \ldots, n-1$.

Once you have figured out why it pays to "speculate" and bid higher than the own valuation, it is easy to interpret the second property. Just keep in mind that a rational bidder may get burned and suffer a loss because the kth highest bid is above the own valuation. As the number of bidders is increased, it becomes more likely that the kth highest bid is close to the own valuation. Therefore, it makes sense to bid more conservatively when the number of bidders is increased, even though this may seem somewhat puzzling at first glance.

Finally, the fourth property suggests that a risk-averse seller should always prefer lower-order kth-price auctions and should most prefer the Dutch auction. While the English auction is always appealing because of its overwhelming strategic simplicity, third- and higher-price auctions are strategically just as complicated as the Dutch auction, and in addition expose the seller to unnecessary risk.

Nobody has ever, to our knowledge, applied a third- or higher-price auction in a single-unit framework. So is this just an intellectual curiosity, useful only to challenge your intuition and technical skills?

Experimental economists have exposed inexperienced subjects to third-price sealed-bid auctions and examined their response to a larger number of bidders. Amazingly, the majority of bidders reduced their bid, just as the theory recommends (see Kagel and Levin (1993)). The authors take great pride in this result; they take it as evidence that inexperienced subjects are not as unsophisticated as is often

[13] Third-price auctions were analyzed for the first time by Kagel and Levin (1993), assuming uniformly distributed valuations.

[14] Hints: Suppose the equilibrium bid function of the kth-price auction is linear in the own valuation, $b^*(v) = \alpha_k v$. Revenue equivalence applies also to the kth-price auction (the proof follows in Section 8.2.5). Therefore, the expected price must be the same as under the English auction: $E[V_{(n-1)}] = E[b^*(V_{(n-k+1)})] = \alpha_k E[V_{(n-k+1)}]$. Compute the two expected values and you have the asserted bid function (8.24). Once you have a candidate for solution, you only need to confirm that it does indeed satisfy the mutual best-response requirement. Notice, however, that this sketch raises the question whether the equilibrium strategy is linear.

suspected by critics of the experimental approach. The particular irony is that, if asked to make a guess, trained theorists tend to come up with the wrong hunch concerning the relationship between the number of bidders and equilibrium bids until they actually sit down and do the computations.

All-Pay Auctions

As a last example consider an auction that is frequently used in charities; we call it the *charity* or *all-pay* auction for lack of a better name. Its peculiar feature is that all bidders are required to actually pay their bid. Just as in a standard auction, the item is awarded to the highest bidder. But unlike in standard auctions, all bidders must pay their bid, even those who are not awarded the item.

Using a procedure similar to the above, the equilibrium bid function is[15]

$$b^*(v) = \frac{n-1}{n} v^n. \tag{8.26}$$

This is in line with the observation that bidders often bid only nominal amounts of money for relatively valuable items.

In order to determine the seller's expected revenue, notice that the seller collects the following expected payment from each bidder:

$$E[b^*(V)] = \frac{n-1}{n} \int_0^1 v^n \, dv = \frac{n-1}{n(n+1)}. \tag{8.27}$$

Take the sum over all bidders, and one obtains $E[nb^*(V)] = (n-1)/(n+1)$, which confirms revenue equivalence.

The crucial feature of charity or all-pay auctions is that they make winners and losers pay. We mention that this is also a feature of the not so charitable seller-optimal auction if bidders are risk-averse (see Matthews (1983))[16] or if bidders are financially constrained (see Laffont and Robert (1996)).

All-pay auctions have also been used to model political lobbying, political campaigns, research tournaments, and job promotion (the last two interpretations view bids as agents' effort).[17] An early example of an all-pay auction under complete information is Posner's (1975) well-known reinterpretation of the social loss of monopoly.

Finally, note that instead of requiring all bidders to pay their own bid, one may instead require them pay the second, third, or kth highest bid. In this sense, it would be more precise to describe the above auctions as "first-price all-pay" auctions.

[15] The probability of winning is the same as under the Dutch auction (see (8.11)). However, since all bids must actually be paid, the equilibrium bid must be a maximizer of $\rho(b)v - b$; hence it must solve the differential equation $\rho' v = 1$. Combining these two building blocks leads to the differential equation $\sigma(b)^{n-1}\sigma'(b) = 1/(n-1)$. Integration entails $(1/n)\sigma(b)^n = [1/(n-1)]b$, and therefore $b^*(v) = [(n-1)/n]v^n$, as asserted.

[16] However, Matthews also showed that the fee should be negative for sufficiently high valuation types.

[17] See, for example, Baye, Kovenock, and de Vries (1993).

Incidentally, the *war-of-attrition* game – used in oligopoly theory to model strategic exit – is essentially a second-price all-pay auction.

Participation Fees

So far we have considered auction rules where all bidders have an incentive to participate because no one can lose. All these auctions were payoff-equivalent. This suggests the following questions: Is there any common auction that gives rise to different equilibrium payoffs, and if so, is it ever desirable to adopt such a deviation from standard auctions?

Many seemingly innocuous modifications of standard auction rules affect bidder participation and drive a wedge between those who participate, typically those with high valuations, and those who exit, typically because their valuation is below a certain critical valuation v_0. Adding such a distortion is profitable to the seller, even though it involves the risk that no sale takes place.

In the following we discuss one particular example: the introduction of a uniform entry or participation fee. The purpose of this exercise is to motivate and explain the notion of a *critical valuation* v_0 below which it does not pay to participate and to show that standard auctions are not seller-optimal, which motivates the search for optimal auction rules.

Example 8.3 (Participation Fee) Suppose the seller requires a participation fee $c \in [0, 1]$ from each bidder in an English auction. Let there be two bidders with uniformly distributed valuations on the support $[0, 1]$.

Then a bidder participates if and only if his valuation is at or above the critical level $v_0 := \sqrt{c}$. The expected-revenue-maximizing entry fee is $c^* = \frac{1}{4}$, and the maximum expected revenue $\frac{5}{12}$. Therefore, for $n = 2$, the participation-fee-augmented English auction is more profitable than the standard English auction and than the optimal take-it-or-leave-it price (see Examples 8.1 and 8.2).

The proof is sketched as follows:

1. In order to compute v_0, consider the marginal bidder with valuation v_0. His cost of participation, c, must be exactly matched by the expected gain from bidding, $\Pr\{\text{winning}\} v_0 - c = 0$. Note that a bidder with valuation v_0 can win only if the rival does not participate because his valuation is below v_0, in which case the price is equal to zero. Therefore, $\Pr\{\text{winning}\} = v_0$, and hence $v_0^2 - c = 0$, or equivalently $v_0 = \sqrt{c}$, as asserted.
2. Notice that the seller collects the entry fee from both bidders if and only if $V_{(1)} \geq v_0$, in which case the price is equal to $V_{(1)}$. The seller has no earnings if and only if $V_{(2)} < v_0$. And finally, only one bidder participates if and only if $V_{(1)} < v_0 \leq V_{(2)}$. Then the price goes all the way down to zero, and the seller's only earning is the entry fee paid by the one and only active bidder. Therefore, the seller's expected profit is equal to

$$\Pi := \Pr\left\{V_{(1)} \geq v_0\right\}\left(E\left[V_{(1)} \mid V_{(1)} \geq v_0\right] + 2c\right)$$
$$+ \Pr\left\{V_{(1)} < v_0 \leq V_{(2)}\right\} c.$$

Of course,

$$
\begin{aligned}
\Pr\left\{V_{(1)} \geq v_0\right\} &= (1-v_0)^2, \\
\Pr\left\{V_{(1)} < v_0 \leq V_{(2)}\right\} &= 2v_0(1-v_0), \\
E\left[V_{(1)} \mid V_{(1)} \geq v_0\right] &= \frac{2}{(1-v_0)^2}\int_{v_0}^{1} v(1-v)\,dv \\
&= \frac{2v_0+1}{3}.
\end{aligned}
$$

Hence, by all of the above,

$$\Pi = \frac{(1-\sqrt{c})(4c+\sqrt{c}+1)}{3}.$$

3. Evidently, Π is strictly concave in c, and $c^* = \frac{1}{4}$ solves the first-order condition $1-2\sqrt{c} = 0$, which completes the proof.

While the introduction of an entry fee raises the sellers' payoff (the optimal c is not equal to zero), we will later show that there are many ways to achieve the seller's maximum revenue. The seller-optimal auction is characterized by a particular critical valuation v_0, which can be implemented in many ways, for example by setting the right minimum bid or the right entry fee.

*8.2.4 An Alternative Solution Procedure**

There are many ways to solve an auction. In the previous subsection you were introduced to the standard solution procedure, which is straightforward though a bit cumbersome. The purpose of this subsection is to raise your virtuosity and exercise a more elegant solution of a first-price or Dutch auction. This solution procedure is also useful in other auction problems, and it breaks the ground for the analysis in Section 8.2.5.

Proposition 8.1 *First-price and Dutch auctions have a unique symmetric equilibrium strategy*

$$b^*(v) = E\left[V_{(n-1)} \mid V_{(n-1)} < v\right] < v. \tag{8.28}$$

Proof We proceed as follows: Assume b^* is strict monotone increasing (which will be confirmed later on). Consider one bidder, and suppose all other bidders play the strategy b^*. Then that bidder must only consider bids from the range $[b^*(0), b^*(1)]$. Therefore, all relevant deviating bids can be generated by bidding the equilibrium strategy b^* as if the valuation were $x \in [0, 1]$ rather than v, which leads to a bid $b^*(x) \neq b^*(v)$.

Let $U(x, v)$ denote that bidder's payoff, and $\rho(x)$ his probability of winning the auction, provided: 1) his true valuation is v, 2) all others play the strategy b^*, and 3) he bids the amount $b^*(x)$. In a first-price or Dutch auction $U(x, v) := \rho(x)\,(v - b^*(x))$ and, due to the monotonicity of b^*, $\rho(x) := \Pr\{b^*(V_{(n-1)}) < b^*(x)\} = \Pr\{V_{(n-1)} < x\} = F(x)^{n-1}$.

The strategy b^* is a symmetric Bayesian Nash equilibrium if and only if it never pays to deviate from the equilibrium bid $b^*(v)$, i.e.,

$$v \in \arg\max_x U(x, v) \qquad \forall v. \tag{8.29}$$

A necessary condition for (8.29) is that for all v

$$0 = \frac{\partial}{\partial x} U(x, v)\big|_{x=v} = \rho'(v)(v - b^*(v)) - \rho(v) b^{*\prime}(v).$$

This condition can be written in the form $v\,d\rho = d(\rho b)$. Since $b^*(0) = 0$, integration yields $b\rho = \int_0^v x\,d\rho$, which implies the assertion $b^*(v) = E[V_{(n-1)} \mid V_{(n-1)} < v]$.

Partial integration of (8.28) gives

$$b^*(v) = v - \int_0^v \left(\frac{F(x)}{F(v)}\right)^{n-1} dx < v,$$

which confirms bid shading. And differentiation gives

$$b^{*\prime}(v) = (n-1)\frac{F'(v)}{F(v)}(v - b^*(v)) > 0,$$

which confirms the assumed monotonicity of b^*.

The necessary conditions are also sufficient if each stationary point is a global maximum. This requirement holds because $U(x, v)$ is increasing in x iff $x < v$ and decreasing if and only if $x > v$, as shown in the appendix on the pseudoconcavity of bidders' payoff functions (Section 8.10). □

8.2.5 General Solution of Symmetric Auction Games

In many noncooperative games it is difficult, if not impossible, to find a closed-form equilibrium solution unless one works with particular functional specifications. The SIPV auction game framework is an exception. In this framework all standard auctions – indeed, all auctions that select the highest bidder as winner – have a unique symmetric equilibrium.

As we show in this subsection, the general solution of SIPV auction games can be found quite easily. The trick is to begin with a more abstract account of auction games, and then solve for equilibrium payoffs, and establish general revenue-equivalence properties before solving for equilibrium strategies.

Once equilibrium payoffs have been determined, one can turn to particular auction games such as the auctions considered in the previous subsection. Knowing payoffs and the details of the considered auction, it is, in most cases a snap to compute the associated equilibrium bid functions.

The advantage of putting the usual solution procedure upside down is twofold: 1) payoffs are determined and revenue equivalence is established for a large array of auction rules (this could never be achieved by solving all conceivable auctions one by one) and 2) the technique is remarkably simple (no need to do dull manipulations of first-order conditions and solve nonlinear differential equations).

To prepare the more abstract account of auction games, we begin with some slightly technical definitions and remarks.

Probability Distribution

Since bidders' valuations are iid random variables, their joint distribution function $\mathcal{F}$ is just the product of their separate distribution functions $F : [0, \bar{v}] \to [0, 1]$, which are in turn identical due to the assumed symmetry. Therefore,

$$\mathcal{F}(v_1, v_2, \dots, v_n) = F(v_1)F(v_2)\cdots F(v_n). \tag{8.30}$$

Strategies

Bidders' strategies are their *bid function* $b : [0, \bar{v}] \to \mathbb{R}_+$ and their *participation rule* $\xi : [0, \bar{v}] \to \{0, 1\}$. Without loss of generality we characterize symmetric equilibria where all bidders play the same strategies. And we assume that all bidders have an incentive to participate, provided their valuation is not below a certain critical value $v_0 \geq 0$, so that $\xi(v) = 1 \iff v \geq v_0$. This reduces strategies to bid functions $b(v)$ and critical valuations $v_0 \in [0, \bar{v}]$.

Representation of Auction Rules

All auction rules can be represented by an *allocation rule* $\hat{\rho}$ and a *payment rule* $\hat{x}$, where $\hat{\rho}(b_1, \dots, b_n)$ is i's probability of winning the auction, and $\hat{x}(b_1, \dots, b_n)$ i's payment to the auctioneer.[18] The advantage of this representation is that, given rivals' strategies, each bidder's decision problem can be viewed as one of choosing through his bid b a *probability of winning* and a *payment.*

Consider one bidder, say bidder 1, and suppose bidders 2 to n bid according to the equilibrium strategy $b^*(v)$. Then the probability of winning and the expected payment are solely functions of the own action b, as follows:

$$\rho(b) := \mathop{E}_{V_2,\dots,V_n}[\hat{\rho}(b, b^*(V_2), \dots, b^*(V_n))], \tag{8.31}$$

$$\mathcal{E}(b) := \mathop{E}_{V_2,\dots,V_n}[\hat{x}(b, b^*(V_2), \dots, b^*(V_n))]. \tag{8.32}$$

Due to the independence assumption, both ρ and $\mathcal{E}$ depend only on the bid b but not *directly* on the bidder's own valuation v.

Payoff Functions

Suppose rivals play the equilibrium strategy $b^*(v)$. Using the $\rho, \mathcal{E}$ representation, the bidders' payoff function is

$$U(b, v) := \rho(b)v - \mathcal{E}(b). \tag{8.33}$$

Equilibrium

The strategies $(b^*(v), v_0)$ are a (symmetric) Bayesian Nash equilibrium if (for $v \geq v_0$)

$$U(b^*(v), v) \geq U(b, v) \qquad \forall b, \tag{8.34}$$

$$U(b^*(v_0), v_0) = 0. \tag{8.35}$$

[18] In principle, payments may be determined by a lottery. Therefore, $\hat{x}_i(\cdot)$ should be viewed as an expected value.

Results

As a first result, we establish the strict monotonicity of the equilibrium bid function.

Lemma 8.1 (Monotonicity) *The equilibrium bid function $b^*(v)$ is strictly monotone increasing.*

Proof The expected utility of a bidder with valuation v who chooses the bid (action) b when all rivals play the equilibrium strategy $b^*(v)$ is (8.33). The equilibrium action $b = b^*(v)$ must be a best reply. Therefore, the *indirect utility* function is

$$U^*(v) := U(b^*(v), v) = \rho(b^*(v))v - \mathcal{E}(b^*(v)). \qquad (8.36)$$

Since $U^*(v)$ is a maximum-value function, the envelope theorem applies, and one has[19]

$$U^{*\prime}(v) = \frac{\partial}{\partial v} U(b^*(v), v) = \rho(b^*(v)). \qquad (8.37)$$

Obviously, $U^*(v)$ is convex.[20] Therefore, $U^{*'}(v) = \rho(b^*(v))$ is increasing in v. And since $\rho(b)$ is increasing in b, it follows that the equilibrium bid function $b^*(v)$ is increasing in v. In view of the fact that the optimal bid cannot be the same for different valuations, the monotonicity is strict.[21] □

An immediate implication of strict monotonicity, combined with symmetry, is:

Lemma 8.2 (Probability of Winning) *The bidder with the highest valuation wins the auction, provided that valuation is above v_0. Therefore, the equilibrium probability of winning is (for $v \geq v_0$)*

$$\rho^*(v) := \rho(b^*(v)) = \Pr\{\textit{rivals' V's are below } v\} = F(v)^{n-1}. \qquad (8.38)$$

This brings us to the amazing payoff equivalence of a large class of auction games.

Proposition 8.2 (Payoff Equivalence) *All auctions that select the highest bidder as winner have a symmetric equilibrium and share the same critical valuation v_0 give rise to the same expected payoffs.*

19 You may wonder whether a second-order condition is satisfied. The answer is yes, as we show in the appendix (Section 8.10).

20 The proof is a standard exercise in microeconomics. Choose $v_1 \neq v_2$, $\hat{v} := \lambda v_1 + (1-\lambda)v_2$. By definition of a maximum, $\rho^*(\hat{v})v_i - \mathcal{E}^*(\hat{v}) \leq \rho^*(v_i)v_i - \mathcal{E}^*(v_i)$, $i \in \{1, 2\}$. Rearrange, and you get $U^*(\hat{v}) \leq \lambda U^*(v_1) + (1-\lambda)U^*(v_2)$, which proves convexity. Make sure that you also understand the intuition of this result. Notice that the bidder could always leave the bid unchanged when v changes, as a fallback. Therefore, U^* must be at or above its gradient hyperplane, which is convexity.

21 Suppose the bid function contains some flat portion in the interval (v_1, v_2). Then a bidder with some valuation from this interval, say v_2, can raise his expected payoff by marginally raising his bid because then he always wins when the second highest valuation is in that same interval.

Proof Suppose $v \geq v_0$. Integrating (8.37), one obtains, using (8.38) and the fact that $U^*(v_0) = 0$,

$$\begin{aligned} U^*(v) &= \int_{v_0}^{v} U^{*\prime}(x)\, dx + U^*(v_0) \\ &= \int_{v_0}^{v} \rho(b^*(x))\, dx \\ &= \int_{v_0}^{v} F(x)^{n-1}\, dx. \end{aligned} \tag{8.39}$$

Next, solve (8.36) for bidders' expected payment, and one finds (for $v \geq v_0$)

$$\begin{aligned} \mathcal{E}^*(v) &:= \mathcal{E}(b^*(v)) \\ &= vF(v)^{n-1} - \int_{v_0}^{v} F(x)^{n-1}\, dx. \end{aligned} \tag{8.40}$$

These results prove that bidders' equilibrium payoff, probability of winning, and expected payments are the same for all permitted auction rules.

Finally, compute the seller's expected revenue Π. The seller earns from each bidder an expected payment $\mathcal{E}^*(v)$. Since he does not know bidders' valuations, he has to take an expected value, which gives

$$\begin{aligned} \underset{V_{(n)} \geq v_0}{E} [\mathcal{E}^*(V)] &= \underset{V_{(n)} \geq v_0}{E} [VF(V)^{n-1} - U^*(V)] \\ &= \frac{1}{n} \underset{V_{(n)} \geq v_0}{E} \left[V_{(n)}\right] - \underset{V_{(n)} \geq v_0}{E} [U^*(V)]. \end{aligned}$$

Sum over all n, and one has the seller's expected revenue

$$\begin{aligned} \Pi(n, v_0) &= n \underset{V \geq v_0}{E} [\mathcal{E}^*(V)] \\ &= \underset{V_{(n)} \geq v_0}{E} \left[V_{(n)}\right] - n \underset{V \geq v_0}{E} [U^*(V)] \qquad (8.41) \\ &= \int_{v_0}^{\bar{v}} v \frac{d}{dv} F(v)^n\, dv \\ &\quad - n \int_{v_0}^{\bar{v}} \left(\int_{v_0}^{v} F(x)^{n-1}\, dx \right) \frac{d}{dv} F(v)\, dv. \end{aligned}$$

Using integration by parts gives

$$\begin{aligned} \int_{v_0}^{\bar{v}} \left(\int_{v_0}^{v} F(x)^{n-1}\, dx \right) \frac{d}{dv} F(v)\, dv &= \int_{v_0}^{\bar{v}} F(v)^{n-1}\, dv - \int_{v_0}^{\bar{v}} F(v)^n\, dv \\ &= \frac{1}{n} \int_{v_0}^{\bar{v}} \left(\frac{d}{dv} F(v)^n \right) \frac{1 - F(v)}{f(v)}\, dv. \end{aligned}$$

Therefore, the seller's expected revenue is

$$\Pi(n, v_0) = \int_{v_0}^{\bar{v}} \left(v - \frac{1 - F(v)}{f(v)} \right) \left(\frac{d}{dv} F(v)^n \right) dv, \tag{8.42}$$

which is also the same for all permitted auction rules. □

Note that (8.41) indicates nicely how the social surplus from trade (the conditional expected value of $V_{(n)}$, conditional on $V_{(n)} \geq v_0$) is divided between bidders and the seller. It follows immediately that this surplus is maximized by setting $v_0 = 0$. Therefore, the considered auction rules are Pareto-optimal if and only if $v_0 = 0$.

As you can see from the ingredients in the proofs, revenue equivalence applies not only to the four standard auctions but to all auctions that select the highest bidder as winner, provided they share the same critical valuation v_0. However, expected payoffs are usually not the same when critical valuations differ. In particular, while standard auctions are payoff-equivalent when the critical valuation is the same, varying the critical valuation gives rise to a different class of payoff-equivalent auctions with a different payoff level. Keep this in mind if you apply revenue equivalence.

Remark 8.1 Note that the factor that appears in the integrand of (8.42),

$$\gamma(v) := v - \frac{1 - F(v)}{f(v)}, \tag{8.43}$$

can be interpreted as "marginal revenue" (see Section 8.1.3) or as a "priority level" (Myerson).[22] In this light it follows immediately from (8.42) that the seller's expected revenue is equal to the expected value of the highest priority level:

$$\Pi(n, v_0) = \underset{V_{(n)} \geq v_0}{E} \left[\gamma(V_{(n)})\right]. \tag{8.44}$$

Proposition 8.3 (Equilibrium Bid Functions I) *Suppose valuations are drawn from the continuous probability distribution function $F(v)$, and the minimum bid is $v_0 \geq 0$. Then the equilibrium bid functions of standard auction games are (for $v \geq v_0$)*

$$b^*(v) = v - \int_{v_0}^{v} \left(\frac{F(x)}{F(v)}\right)^{n-1} dx \qquad \textit{(Dutch auction)}, \tag{8.45}$$

$$b^*(v) = v \qquad \textit{(English auction)}. \tag{8.46}$$

Of course, the English auction has also several asymmetric equilibria, but these are weakly dominated.

Proof In the Dutch auction, the winner pays his bid, and the loser goes free. Therefore, $\mathcal{E}^*(v) = \rho^*(v) b^*(v)$. Solve this for $b^*(v)$, and one has immediately (8.45).

To establish the asserted symmetric equilibrium bidding rule for the English auction, recall that truth telling is a (weakly) dominant strategy. This proves immediately the asserted solution. However, as an exercise we now also derive the symmetric equilibrium strategy using our solution procedure: By (**??**) one has

$$\begin{aligned}\mathcal{E}^*(v) &= \int_{v_0}^{v} y \frac{d}{dy}(F(y)^{n-1})\, dy \\ &= E\left[V_{(n-1:n-1)} \mid V_{(n-1:n-1)} \in [v_0, v]\right] \Pr\left\{V_{(n-1:n-1)} \in [v_0, v]\right\}.\end{aligned}$$

[22] Myerson (1981) shows that, if valuations are drawn from different distributions, the optimal auction awards the item to the bidder with the highest priority level and charges him an amount equal to the lowest valuation that would still assure winning.

In an English auction the winner pays the second highest bid, and only the winner pays; therefore,

$$\mathcal{E}^*(v) = E\left[b^*\left(V_{(n-1:n-1)}\right)\middle|V_{(n-1:n-1)} \in [v_0, v]\right] \Pr\left\{V_{(n-1:n-1)} \in [v_0, v]\right\}.$$

Hence, $b^*(v) = v,\ \forall v \geq v_0$. □

Proposition 8.4 (Equilibrium Bid Functions II) *Suppose valuations are drawn from the continuous probability distribution function $F(v)$. Then the equilibrium bid functions of some of the other auction games analyzed before are (for $v \geq v_0$)*

$$b^*(v) = vF(v)^{n-1} - \int_{v_0}^{v} F(x)^{n-1}\,dx \qquad \textit{(all-pay auction)}, \tag{8.47}$$

$$b^*(v) = v + \frac{k-2}{n-k+1}\frac{F(v)}{f(v)} \qquad \textit{(kth-price auction, } k \geq 2\textit{)}. \tag{8.48}$$

Proof In the all-pay auction all bidders actually pay their bid with certainty, whether they win or lose. Therefore, $b^*(v) = \mathcal{E}(b^*(v))$, and the asserted bid function follows immediately from (**??**). The solution of the third-price auction is a bit more involved and hence omitted (see Wolfstetter (1998)). □

In order to illustrate the effect of the minimum bid upon the bid function in the first-price auction, take a look at Figure 8.3, where we plot the equilibrium bid functions for the case of uniformly distributed valuations on the support $[0, 1]$, and two bidders under $v_0 = 0$ (dashed curve) and $v_0 = \frac{1}{2}$ (solid curve).

Remark 8.2 In a first-price auction the minimum bid p_0 induces the same critical valuation $v_0 = p_0$, which is why we identified v_0 and p_0 in this case. However, the two do not coincide in all auctions. For example, in an all-pay auction the critical valuation v_0 bears the following relationship to the minimum price:

$$v_0 = \frac{p_0}{\rho(v_0)} > p_0,$$

because the bidder with valuation v_0 is indifferent between participation and non-participation if and only if $\rho(v_0)v_0 - p_0 = 0$.

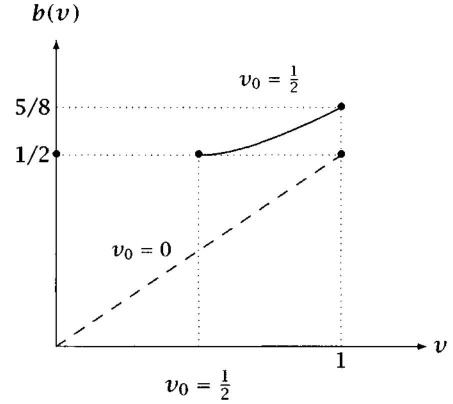

Figure 8.3. Effect of Minimum Price v_0 on Equilibrium Bids in a First-Price Auction.

Also note the following interpretation of equilibrium bidding.

Proposition 8.5 (Interpretation) *In equilibrium each bidder bids so high that his expected payment, conditional upon winning, exactly matches the item's value to rival bidders:*

$$\mathcal{E}^*(v) = \rho^*(v) E\left[V_{(n-1)} \mid V_{(n)} = v\right]. \tag{8.49}$$

Therefore, the equilibrium bid function of the first-price auction can also be written as

$$b^*(v) = E\left[V_{(n-1)} \mid V_{(n)} = v\right] \qquad \textit{(first-price auction)}. \tag{8.50}$$

Finally we mention that the second-order stochastic dominance relationship between the seller's payoff under the Dutch and the English auction, $P_D \succ_{\text{SSD}} P_E$, holds generally and not just for uniformly distributed valuations.

Proposition 8.6 (Stochastic Dominance) *The equilibrium price under the Dutch auction, P_D, second-order stochastically dominates that under the English auction, P_E. Therefore, a risk-averse seller prefers the Dutch over the English auction.*

Proof (Sketch) Suppose the highest valuation is v. Then the conditional cdf's of P_D and P_E, conditional on $V_{(n)} = v$, are single-crossing. Note that the Dutch auction puts all probability mass on $b^*(v)$, whereas under the English auction P_E is determined by the random variable $V_{(n-1)}$. Moreover, $E[P_D \mid V_{(n)} = v] = E[P_E \mid V_{(n)} = v]$ since $\mathcal{E}^*(v \mid V_{(n)} = v)$ is the same under both auctions, by (??). Therefore, for each $V_{(n)} = v$, P_E is a mean-preserving spread of P_D, and P_D, second-order stochastically dominates P_E. Since this applies to all possible highest valuations, it also applies to the average highest valuation. □

*8.2.6 Vickrey Auction as a Clarke–Groves Mechanism**

In his analysis of the efficient provision of public goods, Clarke (1971) proposed his well-known *pivotal mechanism* that implements the efficient allocation in dominant strategies. This mechanism was later generalized by Groves (1973).[23] Interestingly, the Vickrey auction is a special case of the Clarke–Groves mechanism.

The Clarke–Groves mechanism, denoted by (t, k, k'), is defined by a transfer rule t and the allocation rules k and k' that prescribe an allocation and payments, contingent upon agents' valuations v. The allocation rule $k(v)$, $v := (v_1, \ldots, v_n)$, prescribes the allocation that is efficient for the valuations v reported by all agents. And the allocation rules $k'_{-i}(v_{-i})$ prescribe the efficient allocations that would result if agent i were not present, where $v_{-i} := (v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n)$. Finally, the transfer rule stipulates payments to agents if $t_i(v) > 0$ (and from agents if $t_i(v) < 0$),

[23] Green and Laffont (1979) showed that the Clarke–Groves mechanism is the only one that implements the efficient allocation in dominant strategies. They also showed that budget balancing (the requirement that the sum of all transfers vanish) cannot generally be achieved by dominant-strategy mechanisms. However, as shown by d' Aspremont and Gérard–Varet (1979), efficiency and budget balancing can be implemented as a Bayesian Nash equilibrium.

contingent upon agents' reported valuations, as follows:

$$t_i(v) := \sum_{j \neq i} u_j(k(v), v_j) - \sum_{j \neq i} u_j(k'_{-i}(v_{-i}), v_j), \tag{8.51}$$

where u_i denotes i's payoff as a function of the allocation and the own valuation.

Note that $t_i(v) > 0$ represents the positive externality that i exerts upon the welfare of the $n-1$ other agents. It is the difference in welfare of others if agent i is present and if i is absent. In turn, if $t_i(v) < 0$, agent i is required to pay a "tax" equal to the negative externality that he inflicts upon others.

Now apply the Clarke–Groves mechanism to the allocation of a single indivisible unit of a private good. In this case, the allocation rule k stipulates that the good is awarded to the agent with the highest valuation, and that $u_i(k, v_i) = v_i$ if v_i is the highest valuation, and that $u_i(k, v_i) = 0$ otherwise. Therefore, by (8.51),

$$t_i(v) = \begin{cases} -v_{(n-1)} & \text{if } v_i = v_{(n)}, \\ 0 & \text{otherwise,} \end{cases} \tag{8.52}$$

i.e., the agent with the highest valuation is required to pay a transfer equal to the second highest valuation, while all others pay nothing. This shows that, in the present framework, the Clarke–Groves mechanism is precisely the social choice rule implemented by the Vickrey auction.

8.3 Robustness*

Assuming the SIPV framework with risk-neutral agents, we have derived the amazing result that *all* auctions that award the item to the highest bidder and give rise to the same critical valuation below which it does not pay to participate share the same equilibrium payoffs. In other words, if all participants are risk-neutral, many different auction forms – including the four standard auctions – are nothing but irrelevant institutional detail. And if the seller is *risk-averse,* he should favor the Dutch over the English auction (and the English over all higher-order kth-price auctions).

These results are in striking contrast to the apparent popularity of the second-price auction as an oral auction and the first-price auction for sealed bids. Has the SIPV model missed something of crucial importance, or have we missed something at work even in this framework?

A strong argument in favor of the English auction is its strategic simplicity. In this auction bidders need not engage in complicated strategic considerations. Even an unsophisticated bidder should be able to work out that setting a limit equal to one's true valuation is the best one can do. Not so under the Dutch auction. This auction game has no dominant strategy equilibrium. As a result, understanding the game is conceptually more demanding, not to speak of computational problems.

However, if the auction cannot be oral, say, because it would be too costly to bring bidders together at the same time and place, there is a very simple reason why one should favor a first-price sealed-bid auction, even though it is strategically more complicated than a second-price or Vickrey auction.

Second-price auctions can easily be manipulated by soliciting *phantom* bids close to the highest submitted bid. This suggests that second-price sealed bid auctions should not be observed if the seller is profit-oriented.

There are several further good reasons why the seller should favor the Dutch auction. But these come up only as we modify the SIPV model. We now turn to this task. In addition, we will give a few examples of auctions that violate the prerequisites of revenue equivalence and on that ground fail to be payoff-equivalent to the four common auctions.

8.3.1 Introducing Numbers Uncertainty

In many auctions bidders are uncertain about the number of participants (*numbers uncertainty*). This seems almost inevitable in sealed-bid auctions where bidders do not convene in one location but make bids each in their own office.

Notice, however, that the seller can easily eliminate numbers uncertainty if he wishes to do so. He only needs to solicit *contingent* bids where each bidder makes a whole list of bids, each contingent on a different number of participating bidders. Therefore, the seller has a choice. The question is: Is it in the interest of the seller to leave bidders subject to numbers uncertainty?

McAfee and McMillan (1987b) explored this issue. They showed that if bidders are risk-averse and have constant or decreasing absolute risk aversion, numbers uncertainty leads to more aggressive bidding in a first-price closed auction. Since numbers uncertainty obviously has no effect on bidding strategies under the three other auction rules, one can conclude from the revenue equivalence (Proposition 8.2) that numbers uncertainty favors the first-price closed auction. Incidentally, this result is used in experiments to test whether bidders are risk-averse (see Dyer, Kagel, and Levin (1989)).

8.3.2 Discrete Valuations

If valuations are *discrete* random variables, say $V \in \{0, v_1, v_2, \dots, v_r\}$, the Dutch auction has only a mixed-strategy equilibrium. This equilibrium has remarkable properties that make it a truly interesting case and not just a technical side issue.

When you hear that bidders use mixed strategies, you may suspect that efficiency and hence revenue equivalence are sacrificed. By chance, high-valuation bidders may bid low and a low-valuation bidder bid high, in which case the item is not awarded to the highest-valuation bidder.

However, this cannot occur. The equilibrium mixed strategies are continuous probability distribution functions with a remarkable monotonicity property: the supports of bids associated with $v_1 < v_2 < \cdots < v_n$ are

$$[0, b_1], [b_1, b_2], \dots, [b_{n-1}, b_n],$$

where $b_1 < b_2, \dots, < b_n$. Due to this neighboring property of the supports, efficiency is maintained despite randomization, and revenue equivalence prevails.[24]

[24] For an ingeniously simple proof see Riley (1989).

As an illustration, suppose there are only two bidders, and their valuations are either 0 or 1, with equal probability. Then the unique symmetric equilibrium is as follows: if the valuation is equal to 0, do not participate; if it is equal to 1, participate and bid according to the probability distribution function[25]

$$B(x) := \Pr\{\text{bid} < x\} = \frac{x}{1-x}, \qquad x \in [0, \tfrac{1}{2}], \tag{8.53}$$

with $B(0) = 0,\ B(\frac{1}{2}) = 1$.

In a mixed-strategy equilibrium, all actions from the support of the equilibrium mixed strategy, $[0, \frac{1}{2}]$, are an equally good response to $B(x)$. Therefore, bidders' equilibrium payoffs in the Dutch-auction game are $U^*(0) = 0$, $U^*(1) = \frac{1}{2}$.

Finally, consider the English auction. There, bidders stick to the truth-telling pure strategy $b^*(v) \equiv v$. A bidder with valuation $v = 1$ has an equal chance to face a rival either with valuation $v = 0$ or with $v = 1$. If the rival has $v = 1$, the item is sold at price 1 by using some tie rule, and neither bidder gains or loses. And if the rival has $v = 0$, the item is sold at price zero at a gain equal to 1. From this you see immediately that bidders' equilibrium payoffs are the same as in the Dutch-auction game. Finally, since it is always the highest valuation that wins the item, the overall gain from trade and therefore the seller's expected revenue are also the same.

8.3.3 Removing Bidders' Risk Neutrality

A more substantial modification is obtained by introducing risk-averse bidders. Obviously, this modification does not affect the equilibrium strategy under the English auction. Therefore, the expected price in this auction is unaffected as well.

However, strategies and payoffs change under the Dutch auction. Recall that in this auction form bidders take a chance and *shade* their bid below their valuation. When risk aversion is introduced, shading becomes less attractive because the marginal increment in wealth associated with winning the auction at a slightly reduced bid weighs less heavily than the possible loss of not winning due to such a reduced bid. Therefore, a risk-averse bidder shades his bid less than a risk-neutral bidder. Of course, bidders thus raise the seller's payoff and, alas, reduce their own.[26]

Proposition 8.7 *Suppose bidders are risk-averse. Then the seller prefers the Dutch to the English auction.*

8.3.4 Removing Independence: Correlated Beliefs

Finally, remove independence from the SIPV model and replace it by the assumption of a positive correlation between bidders' valuations. Positive correlation (this is

[25] Make sure to check that all bids (actions) $x \in [0, \frac{1}{2}]$ are maximizers of $\frac{1}{2}(1-x)(1+B(x))$, and therefore best replies to $B(x)$.

[26] For a formal proof see Riley and Samuelson (1981). On the design of optimal auctions when bidders are risk-averse see Maskin and Riley (1984b).

a special case of *affiliation*) means that a high own valuation makes it more likely that rival bidders' valuations are high as well.

The main consequence of positive correlation is that it makes bidders bid lower in a Dutch auction. Of course, correlation does not affect bidding in an English auction, where truth telling is always the dominant strategy. Therefore, the introduction of correlation reduces the expected price under the Dutch auction but has no effect on the expected price under the English auction. In other words, correlation entails that the seller should prefer the English to the Dutch auction.

Proposition 8.8 *In the symmetric private-values model with positively correlated valuations, the seller's payoff is higher under the English than under the Dutch auction, and that of potential buyers is lower.*[27]

The intuition for this result is sketched as follows: Each bidder knows that he will win the auction if and only if all other bidders' valuations are lower than his own. The introduction of correlation means that those with low valuations also think it is more likely that rival bidders have low valuations as well. Therefore, bidders with lower valuations shade their bids more. In turn this convinces high-valuation bidders that they do not have to bid so aggressively (for an instructive example see Riley (1989, p. 48 ff.)).

The introduction of correlation has a number of comparative-statics implications with interesting public policy implications. In particular, the release of information about the item's value should raise bids in a Dutch auction. Incidentally, this comparative-statics property was used by Kagel, Harstad, and Levin (1987) to test the predictions of auction theory in experimental settings.

You may return to the analysis of auctions with correlated valuations in Section 8.7, where you will find a more detailed analysis and some simple proofs.

8.3.5 Removing Symmetry

The symmetry assumption is central to the revenue-equivalence result. For if bidders are characterized by different probability distributions of valuations, under the Dutch auction it is no longer assured that the object is awarded to the bidder with the highest valuation. But precisely this property was needed in the proof of Proposition 8.2.

Example 8.4 Suppose there are two bidders, A and B, characterized by their random valuations, with supports $(\underline{a}, \bar{a})$ and $(\underline{b}, \bar{b})$, with $\bar{a} < \underline{b}$. Consider the Dutch auction. Obviously, B could always win and pocket a gain by bidding the amount $\bar{a}$. But B can do even better by shading the bid further below $\bar{a}$. But then the low-valuation bidder A wins with positive probability, violating Pareto optimality.

An important implication is that Pareto optimality is only assured by second-price bids. If a public authority uses auctions as an allocation device, efficiency should be

[27] The formal proof is in Milgrom and Weber (1982).

the primary objective. This suggests that public authorities should employ second-price auctions.

A detailed study of asymmetric auctions is in Maskin and Riley (1996). They state sufficient conditions under which first-price auctions are preferred by the seller. Altogether, their results suggest that the first-price auction tends to generate higher expected revenue when the seller has less information about bidders' preferences than bidders do about each others' preferences. But if the seller is equally well informed and therefore is in a position to set an appropriate minimum price, the ranking is most likely reversed.[28]

Based on these observations, Maskin and Riley suggest that

> ... in procurement bidding, where the procurer almost certainly knows less about the suppliers than they do about each other, the sealed first-price auction is very likely to be preferred by the procurer.

whereas

> ... in art auctions, where the auctioneer is likely to have at least as much information about buyers' reservation prices as each buyer has about his competitors, this presumption in favor of the sealed first-price auction no longer obtains.

8.3.6 Multiunit Auctions

Suppose the seller has more than one unit of a good and auctions them simultaneously. Then everything generalizes quite easily as long as each buyer demands at most one unit. But revenue equivalence generally fails if buyers are interested in buying several units and their demand is a function of price.

Consider the case if each buyer wants at most one unit. First of all we need to generalize the notions of first- and second-price auctions. Suppose k units are put up for sale. In a generalized second-price auction, a uniform price is set at the level of the highest rejected bid. The highest k bidders receive one unit each and pay that price. This shows how the kth-price auctions reviewed earlier are important in a multiunit setting. Of course, in a generalized first-price auction, the highest k bidders are awarded the k items and each pays his own bid. Therefore, the generalized first-price auction involves price discrimination.

Just as in the single-unit case, one can show that all auction rules are revenue-equivalent, provided they adhere to the principle of awarding each item to the highest bidder (see Harris and Raviv (1981)). However, this does not generalize when bidders may buy several units depending on the price.

A simple analysis of price-dependent demand is in Hansen (1988), who showed that first-price auctions lead to a higher expected price, so that revenue equivalence breaks down in this case. An important application concerns industrial procurement

[28] Another study of asymmetric auctions is Plum (1992). However, Plum considers a very special case, focuses on issues of existence and uniqueness, and does not address the ranking of different auctions by their expected revenue.

contracts in private industry and government. Hansen's results explain why procurement is usually in the form of a first-price seal-bid auction. For a fuller analysis of this matter, see Section 8.8.1 on auctions and oligopoly below.

Altogether, the theory of multiunit auctions is not very well developed. Until recently, one has often avoided modeling the auction as a game and instead analyzed bidders' optimal strategy, assuming a given probability distribution of being awarded the demanded items.[29] However, Maskin and Riley (1989) have generalized the theory of optimal auctions to include the multiunit case. And Spindt and Stolz (1989) showed that as the number of bidders or the quantity put up for auction is increased, the expected stopout price (the lowest price served) goes up, which generalizes well-known properties of single-unit auctions.

The most important applications of multiunit auctions are in financial markets. For example, the U.S. Treasury sells marketable bills, notes, and bonds in more than 150 regular auctions per year, using a sealed-bid, multiple-price auction mechanism. Some of the institutional details of the government securities market are sketched in Section 8.8.3 below.

8.3.7 Split-Award Auctions

Consider a procurement problem – say, the government wants to buy a sophisticated piece of military hardware. There are only two potential suppliers. Suppose it is cost-minimizing to break down the production into two subunits and to let each subunit be produced by a different supplier, due to sufficiently strong decreasing returns to scale. Then the government faces a dilemma: on the one hand, it would like to have each supplier produce one subunit because subdivision assures cost efficiency, and on the other hand, it would like to have competition between suppliers, as a safeguard against monopolistic pricing. How can it reconcile both goals?

One answer is to introduce a *split-award auction*. Such an auction asks each supplier to submit two bids: one for the subunit and one for the complete unit. The auctioneer selects those bids that minimize the cost of the complete unit. In equilibrium each supplier is awarded one subunit at a price that is lower than the monopoly price, due to competition between firms for the entire unit. Unfortunately, split-award auctions do not always perform so well (see Anton and Yao (1989, 1992)).

8.3.8 Repeated Auctions

If a seller has several units, he may also sell them sequentially, one unit after the other. This method of sale gives rise to a *repeated-auction* problem.

Sequential auctioning is common, for example, in the sale of wine, race horses, and used cars. In some of these markets it has been observed that prices tend to follow

[29] See for example the analysis of Treasury-bill auctions by Scott and Wolf (1979) and Nautz and Wolfstetter (1997). Treasury-bill auctions are explained in Section 8.8.3.

a declining pattern (Ashenfelter (1989), Ashenfelter and Genovese (1992)). Several explanations have been proposed for this declining-price anomaly.[30]

Ashenfelter suggested that risk aversion among bidders may explain declining prices in sequential auctions of identical objects, such as identical cases of wine or virtually identical condominiums. However, McAfee and Vincent (1993) could only generate declining prices by assuming that bidders' absolute degree of risk aversion is *nondecreasing* – which seems to violate most peoples' attitude towards risk. Therefore, risk aversion alone is probably not the answer.

While there is no compelling reason for declining prices when identical objects are auctioned, declining prices follow easily if each bidder's valuations for the different objects are independently drawn from an identical distribution, and each bidder wants to buy only one unit (see Bernhardt and Scoones (1994)).

To understand why prices fall in a sequential second-price auction of stochastically identical objects, it is useful to compare with the case of identical objects. In his study of a sequential second-price auction of exactly identical objects, Weber (1983) pointed out that bidders should bid less than their valuation in the first round to allow for the option value of participating in subsequent rounds. Since Weber assumed identical objects, bidders with a higher valuation have also a higher option value, and therefore they should shade their bids in the first round by a greater amount than bidders with a lower valuation. This way, all gains to waiting are arbitraged away and, as Weber showed, the expected prices in all rounds are the same.

However, if the objects for sale are only stochastically the same, all bidders have the same option value, regardless of their valuation for the first object put up for sale. For a precise statement of equilibrium suppose there are $n > 3$ bidders and two stochastically identical units sold in a sequence of second-price auctions. Each bidder wants to acquire only one unit.

Bidders' expected gain in the second auction is

$$\xi := \frac{1}{n-1}\left(E\left[V_{(n-1:n-1)}\right] - E\left[V_{(n-2:n-1)}\right]\right). \tag{8.54}$$

Since a bidder could always delay bidding until the second round, and since he is interested in just one unit, ξ is the *option value* of delay.

Another explanation of declining prices is based on the assumption of stochastic scale effects, in the form of either economies or diseconomies of scale (see Jeitschko and Wolfstetter (1998)).

8.4 Auction Rings

So far we have analyzed auctions in a noncooperative framework. But what if bidders collude and form an *auction ring*? Surely, bidders can gain a lot by collusive agreements that exclude mutual competition. But can they be expected to achieve a stable and reliable agreement? And if so, which auctions are more susceptible to collusion than others?

[30] On the role of information transmission and learning in repeated auctions see Jeitschko (1998).

To rank different auctions in the face of collusion, suppose all potential bidders have come to a collusive agreement. They have selected their *designated winner*, who we will assume is the one with the highest valuation, recommended him to follow a particular strategy, and committed others to abstain from bidding. However, suppose the ring faces an enforcement problem because ring members cannot be prevented from breaching the agreement. Therefore, the agreement has to be self-enforcing. Does it pass this test at least in some auctions?

Consider the Dutch auction (or first-price sealed bid). Here, the designated winner will be recommended to place a bid roughly equal to the seller's minimum price, whereas all other ring members are asked to abstain from bidding. But then each of those asked to abstain can gain by placing a slightly higher bid, in violation of the ring agreement. Therefore, the agreement is not self-enforcing.

Not so under the English auction (or second-price closed-seal auction). Here the designated bidder is recommended to bid up to his own valuation and everyone else to abstain from bidding. Now, no one can gain by breaching the agreement, because no one will ever exceed the designated bidder's limit.

Therefore:[31]

Proposition 8.9 *Collusive agreements between potential bidders are self-enforcing in an English but not in a Dutch auction.*[32]

These results give a clear indication that the English auction (or second-price closed auction) is particularly susceptible to auction rings, and that the seller should choose a Dutch instead of an English auction if he has to deal with an auction ring that is unable to enforce agreements.

Even if auction rings can write enforceable agreements, the ring faces the problem of how to select the designated winner and avoid strategic behavior of ring members. This is usually done by running a pre-auction. But can one set it up in such a way that it always selects that ring member with the highest valuation?

In a pre-auction, every ring member is asked to place a bid, and the highest bidder is chosen as the ring's sole bidder at the subsequent auction. But if the bid at the pre-auction only affects the chance of becoming designated winner at no cost, each ring member has every reason to exaggerate his valuation. Therefore, the pre-auction problem can only be solved if one makes the designated winner share his alleged gain from trade.

Graham and Marshall (1987) proposed a simple scheme that resolves the pre-auction problem by an appropriate side-payment arrangement. Essentially, this scheme uses an English auction (or second-price closed auction) to select the designated winner. If the winner of the pre-auction also wins the main auction, he is required to pay the difference between the price paid at the main auction and the second highest bid from the pre-auction bid to a third party. This way, the entire

[31] This point was made by Robinson (1985).

[32] To be more precise, under an English auction the bidding game has two equilibria: one where everybody sticks to the cartel agreement and one where it is breached. The latter can, however, be ruled out on the ground of payoff dominance.

game (pre- plus main auction) is made equivalent to a second-price auction without collusion, which solves the pre-auction problem.

In order to prevent all the gains from collusion from going to the third party, the ring has to sell the entitlement to this payment to an interested party and divide that payment equally, prior to the pre-auction. This way, all ring members capture a share of the gains from collusion.

Alternatively, the ring could arrange the pre-auction without using a third party. For example, the ring could require the winner to pay to all ring members an equal share of the difference between the price paid at the main auction and the second highest bid from the pre-auction. Such auction games have been analyzed by Engelbrecht–Wiggans (1994).

Notice, however, that these solutions work only if the ring can enforce the agreed-upon side payments. In the absence of enforcement mechanisms, auction rings are plagued by strategic manipulation, so that no stable ring agreement may get off the ground.

In their analysis of *weak cartels* McAfee and McMillan (1992) come to the conclusion that the impossibility of transfer payments precludes efficiency. Tacit collusion implies bidding strategies of the following form (for some $\hat{v} \in (0, 1)$):

$$b^*(v) = \begin{cases} 0 & \text{if } v < \hat{v}, \\ \hat{v} & \text{if } v \geq \hat{v}, \end{cases} \tag{8.55}$$

which evidently entails inefficiency.

Incidentally, this tacit collusion strategy has an empirical counterpart. As mentioned by Scherer (1980),

> Each year the Federal and State governments receive thousands of sets of identical bids in the sealed bid competitions they sponsor.

Finally, we mention that the seller may fight a suspected auction ring by "pulling bids off the chandelier," that is, by introducing imaginary bids into the proceedings. In 1985, the New York City Department of Consumer Affairs proposed to outlaw imaginary bids after it had become known that Christie's had reported the sale of several paintings that had in fact not been sold. This proposed regulation was strongly opposed by most New York-based auction houses, precisely on the ground that it would deprive them of one of their most potent weapons to fight rings of bidders.

8.5 Optimal Auctions

Among all possible auctions, which one maximizes the seller's expected profit? At first glance, this question seems unmanageable. There is a myriad of possible auction rules limited only by one's imagination. Whatever your favorite auction rule, how can you ever be sure that someone will not come along and find a better one?

On the other hand, you may ask, what is the issue? Did we not show that virtually all auction rules are payoff-equivalent?

The breakthrough that made the problem of optimal auction design tractable is due to two contributions, one by Riley and Samuelson (1981) and the other by Myerson (1981).

In a superficial interpretation, the two contributions differ in the style of how they approach the optimal-auction problem. Whereas Riley and Samuelson start out with a simple characterization of revenue-equivalence relationships (much of which has entered into our analysis of standard auctions) and then reduce the optimal-auction problem to the optimal choice of minimum price, Myerson poses optimal auctions as a mechanism design problem.

However, the difference goes deeper. The approach by Riley and Samuelson is more restrictive, for two reasons:

- Whereas Riley and Samuelson assume that the item is awarded to the highest bidder, Myerson shows the limitations of this commonly used allocation rule.
- Whereas Riley and Samuelson assume that all bidders' valuations are drawn from the same distribution, Myerson allows them to be drawn from different distributions (asymmetric auctions).

In the following we first review the solution by Riley and Samuelson, which builds directly upon our previous results. Then we explain Myerson's mechanism-design approach. And finally we add some remarks on how optimal-auction results change if one takes into account the cost of participation and stochastic entry.

8.5.1 A Simplified Approach

In the symmetric case it is easy to characterize optimal auctions, without employing the apparatus of the mechanism-design approach if one is ready to restrict attention to the allocation rule awarding the item to the highest bidder. This simplified approach follows Riley and Samuelson (1981). We will later confirm that the assumed allocation rule is not restrictive in the symmetric case. Therefore, the only serious limitation is the symmetry assumption.

For this purpose, go back to our solution of the seller's equilibrium payoff (8.42). There, the only remaining choice variable for the seller is the critical valuation v_0. Therefore, the seller-optimal auction is the one that maximizes $\Pi(n, v_0)$ over v_0.

Proposition 8.10 (Optimal Auctions without Entry) *The seller optimal auction sets the cutoff valuation v_0^* (below which it does not pay to bid) that satisfies the condition*

$$v_0^* = \frac{1 - F(v_0^*)}{f(v_0^*)}, \tag{8.56}$$

independent of the number of bidders n. Therefore, a modified second-price auction – modified by imposing a minimum bid equal to v_0^* – is optimal. For example, if valuations are uniformly distributed on the support $[0, 1]$, *then* $v_0^* = 1/2$ *(Figure 8.4).*

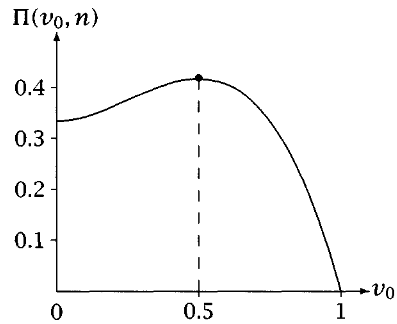

Figure 8.4. Optimal Auction: Uniform Distribution and $n = 2$.

Proof Differentiate $\Pi^*(v_0, n)$ from (8.42) with respect to v_0, and one obtains

$$\frac{\partial}{\partial v_0}\Pi^*(n, v_0) = \varphi(v_0)\frac{\partial}{\partial v_0}F(v_0)^n, \tag{8.57}$$

$$\varphi(v) := \frac{1 - F(v)}{f(v)} - v. \tag{8.58}$$

Note that (8.57) has also a stationary point at $v_0 = 0$. However, since $\Pi^*(v_0, n)$ is strictly increasing for all positive v_0 close to 0, the stationary point at $v_0 = 0$ is a minimum. Therefore, the maximizer must solve (8.56). Existence is assured because $\varphi(0) = 1/f(0) > 0$ and $\varphi(\bar{v}) = -\bar{v} < 0$. We mention that if the *hazard rate* $f(v)/(1 - F(v))$ is strictly monotone increasing, v_0^* is unique.[33]

If you are still somewhat surprised by this result, take a look at the following intuitive argument.

Suppose the minimum price is v_0, and the seller raises it to $v_0 + \delta$. In which case does this matter? The seller gains δ only if he squeezes a single bidder who bids at or above $v_0 + \delta$, which occurs with probability $nF(v_0)^{n-1}(1 - F(v_0 + \delta))$. And the seller loses v_0 if the maximum bid happens to be between v_0 and $v_0 + \delta$, which occurs with probability

$$nF(v_0)^{n-1}\left(F(v_0 + \delta) - F(v_0)\right).$$

Therefore, the net expected gain, per increment in v_0, is

$$\Delta(\delta) := nF(v_0)^{n-1}(1 - F(v_0 + \delta)) - nF(v_0)^{n-1}\frac{F(v_0 + \delta) - F(v_0)}{\delta}v_0.$$

Taking the limit gives

$$\lim_{\delta \to 0}\Delta(\delta) = nF(v_0)^{n-1}\left((1 - F(v_0)) - f(v_0)v_0\right),$$

and setting $\Delta = 0$, one obtains the asserted $v_0^* = [1 - F(v_0^*)]/f(v_0^*)$. □

Remarkably, the optimal minimum bid is independent of the number of bidders.

[33] Otherwise, (8.56) may have multiple roots, in which case one has to evaluate the seller's payoff at all roots, to find out which is the global maximizer.

8.5.2 The Mechanism-Design Approach

We now turn to Myerson's mechanism-design approach. Recall that this approach is more general in that it permits asymmetries and does not impose arbitrary restrictions concerning the allocation rule.

Myerson observed that in the search for optimal auctions one may restrict attention, without loss of generality, to incentive-compatible, direct auctions. This fundamental insight applies the famous *revelation principle*, which is also due to Myerson (1979), and which has been useful in many areas of modern economics, from the public-good problem to the theory of optimal labor contracts.

Direct Auctions

An auction is called *direct* if each bidder is only asked to report his valuation to the seller, and the auction rules select the winner and bidders' payments. Closed auctions are direct auctions; English and Dutch auctions are indirect.

Incentive Compatibility

A direct auction is *incentive-compatible* if honest reporting of valuations is a Nash equilibrium. A particularly strong and strategically simple case is an auction where truth telling is a dominant strategy.

Since we consider optimality from the point of view of the seller, incentive compatibility requires only that buyers reveal their true valuations; we do not require that the seller reports his own true valuation before bids are solicited. Again, the second-price closed auction is a good illustration. Obviously, it is incentive-compatible in the sense that all buyers bid truthfully. But, as we have already learned, it is not truthful with regard to the seller's own valuation because it is in the seller's interest to set a minimum bid above his own valuation.

Revelation Principle

The revelation principle says that *for any equilibrium of any auction game, there exists an equivalent incentive-compatible direct auction that leads to the same probabilities of winning and expected payments.* Therefore, the auction that is optimal among the incentive-compatible direct auctions is also optimal for all types of auctions.

To prove the revelation principle, consider an equilibrium of some arbitrarily chosen auction game. We will show that the following procedure describes an equivalent incentive-compatible direct auction, characterized by an allocation rule ρ and a payment rule x, both contingent on the vector of valuations $v := (v_1, \dots, v_n)$:

1. Ask each bidder to report his valuation.
2. Write down bidders' equilibrium bidding strategies of the assumed auction game, and compute the associated probabilities of winning ρ_i and payments x_i as functions of bidders' valuations v.
3. Set the probabilities of winning $\rho_i(v)$ and payments $x_i(v)$ as in the given equilibrium of the assumed auction game.
4. Select the winner, and collect payments accordingly.

Evidently, if all bidders are honest, this direct auction is equivalent to the given equilibrium of the assumed auction game. Finally, this direct auction is also incentive-compatible. For if it were profitable for any bidder to lie in the direct auction, it would also be profitable for him to lie to himself in executing his equilibrium strategy in the original auction game. It is as simple as that.

Example 8.5 Consider the equilibrium in dominant strategies of the second-price auction. To construct a payoff-equivalent, direct, incentive-compatible auction (ρ, x), set

$$\rho_i(v) = \begin{cases} 1 & \text{if} \quad v_i = \max v, \\ 0 & \text{otherwise,} \end{cases} \tag{8.59}$$

$$x_i(v) = \begin{cases} \max v_{-i} & \text{if} \quad v_i = \max v, \\ 0 & \text{otherwise,} \end{cases} \tag{8.60}$$

where v_{-i} denotes the truncated vector of valuations obtained from v by omitting v_i.

Similarly, the equilibrium of the symmetric first-price auction game can be translated into a payoff-equivalent, incentive-compatible direct auction (ρ, x) by setting ρ as in (8.59) and

$$x_i = \begin{cases} \frac{n-1}{n} v_i & \text{if} \quad v_i = \max v, \\ 0 & \text{otherwise.} \end{cases} \tag{8.61}$$

Feasible Direct Auctions

A direct auction is described by the allocation rule ρ and the payment rule x, both defined on the product set of the support of valuations. Here $\rho_i(v)$ denotes the ith bidder's probability of winning and $x_i(v)$ his payment (notice: the bidder may have to make a payment even if he does not win the auction). To be *feasible*, these functions have to satisfy the following three conditions.

First, because there is only one object to allocate, the probabilities $\rho_i(v_1, \dots, v_n)$ must sum to no more than 1 for all valuations (notice: the sale may fail with positive probability):

$$\sum_{i=1}^{n} \rho_i(v) \leq 1 \qquad \text{(budget constraint).} \tag{8.62}$$

Second, the direct auction has to be incentive-compatible. That is, if a bidder's true valuation is v_i, reporting the truth must be at least as good as reporting any other valuation $\hat{v}_i \neq v_i$. The utility of an agent whose valuation is v_i but who reports the valuation $\hat{v}_i$ is

$$\begin{aligned} U_i(\hat{v}_i \mid v_i) := \mathrm{E}\,[\rho_i(V_1, \dots, \hat{v}_i, \dots, V_n)v_i] \\ -\mathrm{E}\,[x_i(V_1, \dots, \hat{v}_i, \dots, V_n)]. \end{aligned} \tag{8.63}$$

(The V's are random variables; v_i and $\hat{v}_i$ particular realizations.) Therefore, incentive compatibility requires that, for all $v_i, \hat{v}_i \in [\underline{v}_i, \bar{v}_i]$,

$$U_i(v_i \mid v_i) \geq U_i(\hat{v}_i \mid v_i) \qquad \text{(incentive compatibility).} \tag{8.64}$$

Third, participation must be voluntary, i.e., each bidder must be assured a non-negative expected utility,

$$U_i(v_i \mid v_i) \geq 0 \quad \forall v_i \in [\underline{v}_i, \bar{v}_i] \qquad \text{(participation constraint).} \tag{8.65}$$

The Programming Problem

In view of the revelation principle and the above feasibility considerations, the optimal auction design problem is reduced to the choice of (ρ, x) in such a way that the seller's expected profit is maximized (recall that r is the seller's own valuation):

$$\max_{\{\rho_i, x_i\}} \sum_{i=1}^{n} \mathrm{E}\,[x_i(V_1, \ldots, V_n)] - \sum_{i=1}^{n} \mathrm{E}\,[\rho_i(V_1, \ldots, V_n)]r, \tag{8.66}$$

subject to the budget (8.62), the incentive compatibility (8.64), and the participation constraints (8.65). What looked like an unmanageable mechanism design problem has been reduced to a straightforward optimization problem.

In the following we review and interpret the solution. If you want to follow the mathematical proof you have to look up Myerson's (1981) contribution.

Solution

In order to describe the solution, the so-called *priority levels* associated with a bid play the pivotal role. Essentially, these priority levels resemble and play the same role as individual customers' marginal revenue in the analysis of monopolistic price discrimination.

The priority level associated with valuation v_i is defined as

$$\gamma_i(v_i) := v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}. \tag{8.67}$$

We assume that priority levels are monotone increasing in the v_i's.[34]

Going back to the Cournot monopoly problem under incomplete information, reviewed in the introduction (see (8.3)), $\gamma_i(v_i)$ can be interpreted as the marginal revenue from offering the item for sale to buyer i at a take-it-or-leave-it price equal to $p = v_i$. Therefore, the monotonicity assumption means that marginal revenue is diminishing in "quantity," that is, in probability of sale.[35] Of course, the priority level is always less than the underlying valuation v_i.

With these preliminaries, the optimal auction is summarized by the following rules. The first set of rules describes how one should pick the winner, and the second how much the winner shall pay.

[34] Notice $[1 - F_i(v_i)]/f_i(v_i)$ is the reciprocal of the so-called *hazard rate*. Therefore, the assumed monotonicity of priority levels is satisfied whenever the hazard rate is not decreasing. A sufficient condition is obviously the log concavity of the complementary distribution function $\bar{F}(v) := 1 - F(v)$. ($\bar{F}$ is called log concave if $\ln \bar{F}$ is concave.) This condition holds, for example, if the distribution function F is uniform, normal, logistic, chi-squared, exponential, or Laplace. See Bagnoli and Bergstrom (1989).

[35] Myerson (1981) also covers the case when priority levels are not monotone increasing.

Selection of Winner

1. Ask each bidder to report his valuation, and compute the associated priority levels.
2. Award the item to the bidder with the highest priority level, unless it is below the seller's own valuation r.
3. Keep the item if all priority levels are below the seller's own valuation r (even though the highest valuation may exceed r).

Pricing Rule To describe the optimal pricing rule we need to generalize the second-price auction rule. Suppose i has the valuation v_i, which leads to the highest priority level $\gamma_i(v_i) \geq r$. Now ask: By how much could i have reported a lower valuation and still have won the auction under the above rules?

Let z_i be the lowest reported valuation that would still lead to winning,

$$z_i := \min\{\tilde{v} \mid \gamma_i(\tilde{v}) \geq r,\ \gamma_i(\tilde{v}) \geq \gamma_j(v_j),\ \forall j \neq i\}. \tag{8.68}$$

Then the optimal auction rule says that the winner shall pay z_i. This completes the description of the auction rule.

Interpretation Essentially, the optimal auction rule combines the idea of a second-price (or Vickrey) auction with that of third-degree monopolistic price discrimination.[36] As a first step, customers are ranked by their marginal revenue $v_i - [1 - F_i(v_i)]/f(v_i)$, evaluated at the reported valuation. Since only one indivisible unit is up for sale, only one customer can win. The optimal rule selects the customer who ranks highest in the marginal-revenue hierarchy, unless it falls short of the seller's own valuation r. In this sense the optimal selection of the winner rule employs the techniques of third-degree price discrimination.

But the winner is asked to pay neither his marginal revenue nor his reported valuation. Instead, the optimal price is equal to the lowest valuation that would still make the winner's marginal revenue equal to or higher than that of all rivals. In this particular sense, the optimal auction rule employs the idea of a "second-price" auction.

Illustrations

As an illustration, take a look at the following two examples. The first one shows that the optimal auction coincides with the second-price sealed-bid auction supplemented by a minimum price above the seller's own valuation, if customers are indistinguishable. The second example then brings out the discrimination aspect, assuming that bidders are different.

In both examples, the seller deviates from the principle of selling to the highest bidder (which underlies revenue equivalence), and the equilibrium outcome fails to be Pareto-optimal with positive probability. This indicates that the four standard auctions are not optimal. However, the optimal auction itself poses a credibility problem that will be discussed toward the end of this section.

[36] Recall that in the theory of monopoly one distinguishes between three kinds of price discrimination: perfect or *first-degree* price discrimination, imperfect or *second-degree* price discrimination by means of self-selection devices (offering different price–quantity packages), and finally *third-degree* price discrimination based on different signals about demand (for example, different national markets).

Example 8.6 (Identical Bidders) Suppose valuations are uniformly distributed on $[0, 1]$. Then the priority levels are

$$\gamma_i(v_i) = v_i - \frac{1 - v_i}{1} = 2v_i - 1,$$

and a sale occurs only if the highest valuation exceeds $\frac{1}{2} + r/2$. The winner pays either $\frac{1}{2} + r/2$ or the second highest bid, whichever is higher. The optimal auction is equivalent to a modified second-price sealed-bid auction where the seller sets a minimum price equal to $\frac{1}{2} + r/2$. Since that minimum price is higher than the seller's own valuation, the optimal auction is not Pareto-efficient.

With two bidders and a zero seller's own valuation the optimal minimum price is $\frac{1}{2}$, and the expected revenue is equal to $\frac{5}{12}$.[37] Compare this with the Cournot approach, the English auction, and the participation-fee-augmented English auction (see Examples 8.1, 8.2, 8.3).

Example 8.7 (Bidder Heterogeneity) Suppose two bidders, A and B, have uniformly distributed valuations; A's support is $[0, 1]$ and B's $[0, 2]$. Set $r = 0$. Then the priority levels are

$$\gamma_A(v_A) = 2v_A - 1, \qquad \gamma_B(v_B) = 2v_B - 2.$$

Obviously, B can only win if his valuation exceeds that of A by $\frac{1}{2}$. Thus, the optimal auction discriminates against B in order to encourage more aggressive bidding. Note that if B were not discriminated against, he would never report a valuation greater than 1 and never pay more than 1, whereas in the optimal auction he might have to pay up to $\frac{3}{2}$.

Remark

In the symmetric case, the optimal auction can be implemented by a modified second-price auction. This is, however, not the only way. In fact, the optimum can be implemented by any mechanism that awards the item to the highest valuation that exceeds the optimal critical valuation. The list of possible auctions includes first-price, second-price, and even all-pay auctions, supplemented by an optimal minimum price.

*The Case of Two Bidders and Two Valuations**

This brief introduction to optimal auctions has stressed essential properties and left out proofs and computations. This may leave you somewhat unsatisfied. Therefore, we now add a full-scale example with two identical bidders and two valuations.[38] Its main purpose is to exemplify the computation of optimal auctions in a particularly simple case where the price-discrimination aspect of optimal auctions cannot play any role. (See, however, the closing remarks in this sub-subsection.)

[37] The expected revenue is $\Pr\{V_{(1)} \geq \frac{1}{2}\}E[V_{(1)} \mid V_{(1)} \geq \frac{1}{2}] + \frac{1}{2}\Pr\{V_{(1)} < \frac{1}{2} \leq V_{(2)}\}$, with $E[V_{(1)} \mid V_{(1)} \geq \frac{1}{2}] = \frac{2}{3}$, and $\Pr\{V_{(1)} \geq \frac{1}{2}\} = \frac{1}{4}$, $\Pr\{V_{(1)} < \frac{1}{2} \leq V_{(2)}\} = \frac{1}{2}$. As an alternative, just plug $n = 2$ and $F(v) = v$ into (8.42).

[38] The example is borrowed from Binmore (1992, pp. 532–536).

Suppose the two bidders are identical in the sense that they have either a low (v_l) or a high valuation (v_h). These valuations occur with the probabilities $\Pr\{V = v_l\} = \pi, \Pr\{V = v_h\} = 1 - \pi$.

In the framework of this example, a direct auction will be described by 1) the conditional probabilities $\rho_h := E[\rho(V, v_h)]$, $\rho_l := E[\rho(V, v_l)]$ of obtaining the item, and 2) the bidders' expected payments $\mathcal{E}_h := E_V[x(v_h, V]$, $\mathcal{E}_l := E_V[x(v_l, V]$.

Optimization Problem The optimal auction maximizes the expected gain from each bidder,

$$\max_{\rho_h,\rho_l,\mathcal{E}_h,\mathcal{E}_l} G := (1-\pi)\mathcal{E}_h + \pi\mathcal{E}_l, \tag{8.69}$$

subject to the *incentive compatibility*

$$\rho_h v_h - \mathcal{E}_h \geq \rho_l v_h - \mathcal{E}_l, \tag{8.70}$$

$$\rho_l v_l - \mathcal{E}_l \geq \rho_h v_l - \mathcal{E}_h, \tag{8.71}$$

and *participation* constraints

$$\rho_h v_h - \mathcal{E}_h \geq 0, \tag{8.72}$$

$$\rho_l v_l - \mathcal{E}_l \geq 0. \tag{8.73}$$

An immediate implication of incentive compatibility is the monotonicity property

$$\rho_h \geq \rho_l, \quad \mathcal{E}_h \geq \mathcal{E}_l. \tag{8.74}$$

Restrictions due to Symmetry Having stated the problem, we now make it more user-friendly. The key observation is that symmetry implies three further restrictions on the probabilities ρ_h, ρ_l.

From the seller's perspective each bidder wins the item with probability $(1-\pi)$ $\rho_h + \pi\rho_l$; in a symmetric equilibrium this probability cannot exceed $\frac{1}{2}$. Therefore,

$$(1-\pi)\rho_h + \pi\rho_l \leq \frac{1}{2}. \tag{8.75}$$

Also, in a symmetric equilibrium type h must lose with probability $\frac{1}{2}$ or more whenever he faces a type-h rival. Therefore, $1 - \rho_h \geq \frac{1}{2}(1-\pi)$, or equivalently

$$\rho_h \leq 1 + \frac{1}{2}(1-\pi) = \frac{1}{2}(\pi + 1). \tag{8.76}$$

And similarly, type l must lose with probability $\frac{1}{2}$ or more when he faces a type-l rival, so $1 - \rho_l \geq \frac{1}{2}\pi$, or equivalently

$$\rho_l \leq 1 - \frac{1}{2}\pi. \tag{8.77}$$

Combining these restrictions, the set of feasible probabilities ρ_h, ρ_l is illustrated in Figure 8.6. Similarly, the set of feasible expected payments $\mathcal{E}_h, \mathcal{E}_l$ is illustrated in Figure 8.5. In both diagrams, the dashed lines represent the seller's indifference curves.

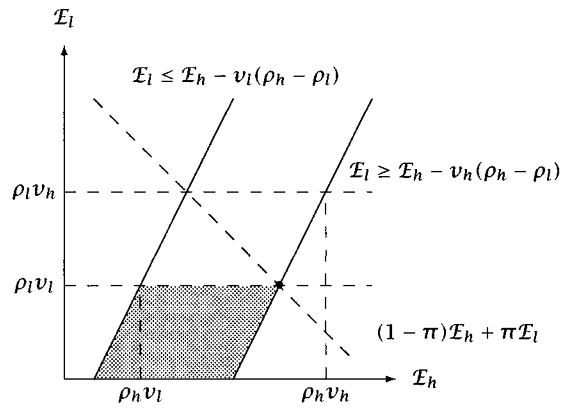

Figure 8.5. Optimal Auction: Optimal $\mathcal{E}$'s.

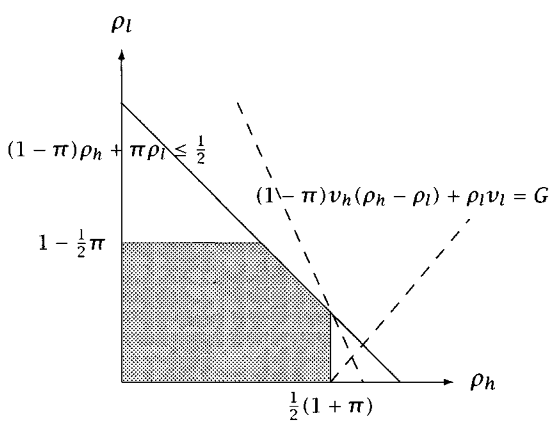

Figure 8.6. Optimal Auction: Optimal ρ's.

Why Some Constraints Do Not Bind As you can see immediately from Figure 8.5, only two constraints are binding: the truth-telling condition concerning type h and the participation constraint concerning type l. Therefore, you can ignore inequalities (8.71), (8.72) and replace inequalities (8.70), (8.73) by equalities.

User-Friendly Optimization Problem Using these results, you can now eliminate the expected payment variables and find the optimal auction as the solution of the following linear programming problem:

$$\max_{\rho_h, \rho_l} G := (1-\pi)v_h(\rho_h - \rho_l) + \rho_l v_l, \tag{8.78}$$

subject to (8.75)–(8.77).

The only variables are ρ_h, ρ_l.

Solution The solution is easily characterized by the two diagrams. Take a look at the seller's indifference curves (the dashed lines) in Figure 8.6. Evidently, the marginal rate of substitution can be positive or negative, depending on the prior probability π. In particular, if type l is sufficiently likely ($\pi \ge (v_h - v_l)/v_h$), the marginal rate of substitution is negative (the dashed indifference curve slopes downwards), whereas

Table 8.2. *Optimal auction*

π	ρ_l	$\mathcal{E}_l$	ρ_h	$\mathcal{E}_h$
$\pi \leq \frac{v_h - v_l}{v_h}$	0	0	$\frac{1}{2}(1+\pi)$	$\frac{1}{2}(\pi + 1)v_h$
$\pi > \frac{v_h - v_l}{v_h}$	$\frac{1}{2}\pi$	$\frac{1}{2}\pi v_l$	$\frac{1}{2}(1+\pi)$	$\frac{1}{2}(v_h + \pi v_l)$

if it is sufficiently unlikely ($\pi \leq (v_h - v_l)/v_h$), the marginal rate of substitution is positive (the dashed indifference curve slopes upwards). Therefore, the solution is at one of two corners of the set of feasible ρ's, depending upon the size of the prior probability π, as summarized in Table 8.2.

Remarks While this example highlights some aspects of optimal auctions, it has also some peculiar properties. In our general statement of the optimal-auction problem, we stressed that optimal auctions are standard auctions supplemented by a minimum-price requirement. However, this holds true only if valuations are continuous random variables. With discrete valuations, as in the present example, the optimal auction is typically not a standard auction.

To see why a standard auction is not optimal in the present example, take a look at the case $\pi > (v_h - v_l)/v_h$. Clearly, if one tries to implement the optimal auction by a second-price sealed-bid auction supplemented by a minimum-price requirement, the optimal $\mathcal{E}_l$, stated in the second row of Table 8.2, requires a minimum price equal to $p = v_l$. However, with this minimum price one cannot also implement the optimal value for $\mathcal{E}_h$.

8.5.3 Secret Reservation Price?

In many auctions the seller keeps his reservation price secret. Secrecy is often justified as a means to fight auction rings. But can a secret reservation price still raise the seller's revenue, or does it destroy the value of commitment?

Consider a first-price auction. If the seller commits to a reservation price and announces it to bidders, he thus induces more aggressive bidding (see Figure 8.3 in Section 8.2.5). The catch is that the seller runs the risk that no transaction takes place, even if the bidders' highest valuation is above the seller's valuation. Therefore, the seller would like bidders to believe that he is committed to a certain reservation price above his own valuation; however, once they are convinced, the seller would most like to revoke that reservation price and not forgo profitable transactions. This suggests that secrecy makes the power of commitment useless, as in the commitment-and-observability problem discussed in Section 3.3.3.

Proposition 8.11 *Consider a first-price auction, and suppose the seller keeps his reservation price secret. Then the equilibrium reservation-price strategy is to set it equal to the seller's own valuation, $\hat{v}$, for all $\hat{v}$.*

Proof In a first-price auction setting the reservation price p_0 induces the critical valuation $v_0 = p_0$. Therefore, we save notation by identifying the reservation price with v_0.

If the seller with valuation $\hat{v}$ sets the secret reservation price $v_0 \geq \hat{v}$, his expected profit is a function of v_0 and $\hat{v}$, as follows (to simplify notation, define $G(y) := \Pr\{V_{(n)} < y\} = F(y)^n, g(y) := G'(y)$):

$$\begin{aligned}\pi(v_0, \hat{v}) &= \hat{v} + \Pr\{b^*(V_{(n)}) \geq v_0\}\left(E\left[b^*(V_{(n)}) \mid b^*(V_{(n)}) \geq v_0\right] - \hat{v}\right)\\ &= \hat{v} + \int_{b^{*-1}(v_0)}^{1} (b^*(y) - \hat{v})\, dG(y).\end{aligned} \tag{8.79}$$

Due to the strict monotonicity of b^*,[39]

$$\begin{aligned}\frac{\partial \pi}{\partial v_0} &= (\hat{v} - b^*(b^{*-1}(v_0)))g(b^{*-1}(v_0))b^{*-1'}(v_0)\\ &= (\hat{v} - v_0)g(b^{*-1}(v_0))\frac{1}{b^{*'}(v_0)}.\end{aligned} \tag{8.80}$$

Therefore, the optimal minimum bid is $v_0 = \hat{v}$. □

However, if the seller runs a second-price or Vickrey auction, secrecy does not hurt him, because in a second-price or Vickrey auction truthful bidding is a weakly dominant strategy, no matter what reservation price is announced. Therefore, the reservation price poses neither a commitment problem nor a commitment-and-observability problem.

The bottom line is that the implementation of the optimal auction requires neither the ability to commit to the optimal reservation price nor its perfect observability, provided the seller adopts a second-price or Vickrey auction. If the seller adopts a first-price or Dutch auction, secrecy completely destroys the power of commitment.

*8.5.4 Optimal Auctions with Stochastic Entry**

We close the discussion of optimal auctions with some results concerning the relationship between the minimum price and bidder participation.

The standard analysis of optimal auctions ignores the problem of entry in the face of costly participation. We now show that optimal auctions differ drastically if this entry problem is taken into account.[40] Recently, Levin and Smith (1994) explored whether one should use a minimum price or entry fees when participation is costly. Here we present a simple and straightforward proof of one of their main assertions.[41]

Suppose bidders have to spend resources to inspect the item for sale, before they know their own valuation. Once the own valuation is known, the cost of inspection, c, is sunk and does not affect further bidding behavior. In this sense, inspection constitutes entry into the bidding game.

[39] Note: since the reservation price is secret, bidders cannot react to it; therefore, bidders' strategy is independent of v_0.

[40] A simple complete-information model of auctions with stochastic entry is in Elberfeld and Wolfstetter (1999). There it is shown that more potential competition lowers the seller's expected revenue, which implies that the seller should restrict the number of bidders.

[41] Incidentally, Tan (1992) analyzed a similar entry problem where bidders in a procurement auction invest in R&D in order to learn their production cost. That game has reasonable pure-strategy equilibria; but, again, the seller should not impose a price ceiling.

The optimal auction problem is now posed as a three-stage game. At *stage 1*, the seller selects an auction rule from the class of auctions considered in previous sections. At *stage 2*, each of m potential bidders decides whether to enter. Entry entails the fixed cost c. Finally, at *stage 3*, the number of active bidders, $n \leq m$, who have entered becomes common knowledge, and they play the usual bidding game.

The number of potential bidders, m, is taken to be so large that all bidders suffer a loss if they all enter, $E[U^*(V; m, v_0)] < c$. If this condition fails, entry is not an issue.

At stage 3 equilibrium payoffs are as shown in Proposition 8.2. Therefore, at stage 2, when bidders do not yet know their own valuation, their reduced-form payoff as a function of n and v_0 is $E[U^*(V; n, v_0)] - c$. Note that $E[U^*(V; n, v_0)]$ is strictly monotone decreasing in n and v_0, and positive (negative) for low (high) n.

Now consider the entry subgame that begins at stage 2. Denote the (mixed) entry strategy by $q := \Pr\{\text{entry}\}$. Then the equilibrium strategy $q^* \in [0, 1]$ must solve the condition of indifference between entry and nonentry,

$$\sum_{n=0}^{m-1} p_{n:m-1}(q) E[U^*(V; n+1, v_0)] - c = 0. \tag{8.81}$$

Here $p_{n:m-1}(q)$ denotes the probability that n rival firms enter (out of a sample of $m-1$),

$$p_{n:m-1}(q) := \binom{m-1}{n} q^n (1-q)^{m-n-1}, \qquad m > n. \tag{8.82}$$

Note that as bidders randomize their entry decision, the number of active bidders becomes a random variable, denoted by the capital letter $0 \leq N \leq m$. Its probability distribution is a binomial distribution, with the following property that will be used repeatedly.

Lemma 8.3 (Stochastic Dominance (FSD)) *When the probability of entry, q, is increased, the random number of bidders, N, increases in the sense of first-order stochastic dominance (FSD). The same applies to an increase in m.*

Proof By a known result, one can write the (complementary) binomial probability distribution function, for $k \in \mathbb{N}$, $0 < k \leq m$,[42]

$$\bar{\Phi}(k; q, m) := \Pr\{N \geq k\} = \sum_{n=k}^{m} p_{n:m}(q) \tag{8.83}$$

$$= \frac{m!}{(k-1)!(m-k)!} \int_0^q \tau^{k-1}(1-\tau)^{m-k}\, d\tau. \tag{8.84}$$

The asserted FSD relationship follows immediately: $q_2 > q_1 \implies \bar{\Phi}(k; q_2, m) > \bar{\Phi}(k; q_1, m)$. □

[42] This formula is proved by repeated integration by parts. See Fisz (1963), equations (10.3.7), (10.3.8).

Proposition 8.12 (Equilibrium Entry Strategy) *The entry subgame has a unique symmetric equilibrium strategy $q^* \in (0, 1)$ that is strictly monotone decreasing in v_0.*

Proof At $q = 0$ the left-hand side (LHS) of (8.81) is positive, and at $q = 1$ negative. Since $E[U^*(V; n, v_0)]$ is strictly monotone decreasing in n, the FSD relationship established in Lemma 8.3 implies that the LHS of (8.81) is strictly monotone decreasing in q. By continuity, there must be a q^* where the indifference condition holds with equality. The asserted comparative statics also follows from Lemma 8.3. □

Using these preliminary results we now turn to the stage 1 subgame and solve the seller optimal auction. For this purpose, use the seller's reduced-form payoff function

$$G(v_0, m) := \sum_{n=1}^{m} p_{n:m}(q^*(v_0))\Pi^*(n, v_0). \tag{8.85}$$

Note that $\Pi^*(n, v_0)$ is increasing in n, and it has a maximum in v_0, which is independent of n, by Proposition 8.10.

Proposition 8.13 (Optimal Auctions with Stochastic Entry) *In the face of stochastic entry, the seller optimal auction sets a minimum price below the static optimum. The optimal auction moves towards the unmodified standard second-price auction, with $v_0^* = 0$.*

Proof Compute the derivative of the seller's expected payoff $G(v_0, m)$ with respect to v_0:

$$\frac{\partial G}{\partial v_0} = \sum_{n=1}^{m} p'_{n:m} q^{*\prime} \Pi^* + \sum_{n=1}^{m} p_{p_{n:m}} \frac{\partial \Pi^*}{\partial v_0}. \tag{8.86}$$

Evaluated at or above the static optimum v_0^*, one has

$$\frac{\partial}{\partial v_0} G(v_0, m) < 0,$$

since $\partial \Pi^* / \partial v_0 \leq 0$, $q^{*\prime} < 0$, and an increase in q "increases" the random number of bidders in the sense of FSD. This shows that the optimal minimum bid is lower than the static optimum. By similar reasoning one can see that $\frac{\partial}{\partial v_0} G(v_0, m) < 0$ also at $v_0 = 0$ (recall: $\Pi^*(v_0, n)$ always has a stationary point at $v_0 = 0$). Therefore, it may very well be optimal to set $v_0 = 0$. □

Remark on Deterministic Entry Consider (one of the many) alternative pure strategy equilibria, where bidders enter until their expected payoff has vanished.[43] Interestingly, even in this framework, no minimum price should be imposed.

[43] Since n is an integer, the precise equilibrium number of bidders is the largest n^* for which $E[U^*(V; n^*)] \geq c$. These equilibria are meaningful if bidders enter sequentially and learn about how many have already entered. They were studied by McAfee and McMillan (1987c) (see also Engelbrecht–Wiggans (1993)).

8.6 Common-Value Auctions and the Winner's Curse

In a famous experiment, Bazerman and Samuelson (1983) filled jars with coins, and auctioned them off to MBA students at Boston University. Each jar had a value of \$8, which was not made known to bidders. The auction was conducted as a first-price sealed-bid auction. A large number of identical runs were performed. Altogether, the *average bid* was \$5.13; however, the *average winning bid* was \$10.01. Therefore, the average winner suffered a loss of \$2.01; alas, the winners were the ones who lost wealth ("winner's curse").

What makes this auction different is just one crucial feature: the object for sale has an *unknown common* value rather than *known private* values. The result is typical for common-value auction experiments. So what drives the winner's curse; and how can bidders avoid falling prey to it?

Most auctions involve some common-value element. Even if bidders at an art auction purchase primarily for their own pleasure, they are usually also concerned about eventual resale. And even though construction companies tend to have private values, for example due to different capacity utilizations, construction costs are also affected by common events, such as unpredictable weather conditions, the unknown difficulty of specific tasks, and random input quality or factor prices. This suggests that a satisfactory theory of auctions should cover private as well as unknown common value components.

Before we consider combinations of private and common value components, we now focus on pure common-value auctions. In a pure common-value auction, the item for sale has the same value for each bidder (e.g., the jar in the above experiment contains the same number of coins for each bidder). At the time of the bidding, this common value is unknown. Bidders may have some imperfect estimate (everyone has his own rule of thumb to guess the number of coins in the jar); but the item's true value is only observed after the auction has taken place.

To sketch the cause of the winner's curse, suppose all bidders obtain an unbiased estimate of the item's value. Also assume bids are an increasing function of this estimate. Then the auction will select the one bidder as winner who received the most optimistic estimate. But this entails that the average winning estimate is higher than the item's value.

To play the auction right, this *adverse selection bias* must already be allowed for at the bidding stage by shading the bid. Failure to follow this advice will result in winning bids that earn less than average profits or even losses.

8.6.1 An Example: The Wallet Auction

An instructive example is the following *wallet auction*, which was introduced by Klemperer (1998): Ask each of two students to check the contents of his wallet without revealing it to the other; then, sell a sum of money equal to the contents of both wallets to the same two students in an English auction.

Each bidder $i = 1, 2$ knows the contents of his own wallet x_i, but neither one knows the sum $v := x_1 + x_2$. Therefore, the item sold has a common value, which is, however, unknown at the time of bidding and only becomes known after the auction, when the contents of both wallets have been revealed.

Proposition 8.14 (Wallet Auction) *The wallet auction has a symmetric equilibrium, described by the strategy* $b^*(x) = 2x$.

Proof Suppose rival j plays the candidate equilibrium strategy b^*. Then, if rival j quits bidding at price p, bidder i updates his valuation to

$$v_i := E[x_i + X_j \mid b^*(X_j) = p] = x_i + \frac{p}{2}.$$

In order to maximize his payoff, bidder i should stay active as long as $v \geq p$, i.e., as long as $p \leq 2x_i$. Hence, the optimal limit at which i should quit is $b^*(x_i) := 2x_i$. In other words, $b^*(x_i)$ is a best reply to $b^*(x_j)$, and vice versa. □

How can one interpret this equilibrium, and how do bidders avoid the winner's curse? When the price is at the level p, bidder i knows that the wallet of rival j must contain at least $p/2$. Therefore, the conditional expected value v_i, conditioned on the event that $p = 2x_i$ has been reached, is equal to or greater than $2x_i$. But then, why should i quit bidding at $p = 2x_i$, and not bid higher?

Here the selection bias enters. If bidder i contemplates whether he should quit bidding at $p = 2x_i$, he must ask himself what is the conditional expected value v_i, conditional on the event that he wins at $p = 2x_i$; because bidder j just quit bidding at this price. Since j plays the strategy $b^*(x_j) = 2x_j$, his quitting at $p = 2x_i$ entails $x_j = x_i$ and hence $v_i = x_i + x_j = 2x_i$. This explains why bidder i should bid neither more nor less than $2x_i$.

We conclude that in a symmetric equilibrium bidder i bids exactly as if rival j had the same amount of money in his wallet, i.e.,

$$b^*(x) = E[X_i + X_j \mid X_i = X_j = x].$$

Incidentally, the wallet auction has also a continuum of asymmetric equilibria, which have entirely different properties. Indeed, the following strategies are an asymmetric equilibrium for all $\alpha > 1$ (of course, the symmetric equilibrium is a obtained for $\alpha = 2$):

$$b_1^*(x_1) = \alpha x_1, \qquad b_2^*(x_2) = \frac{\alpha}{\alpha - 1} x_2.$$

These asymmetric equilibria strongly favor one bidder, to the seller's disadvantage.

8.6.2 Some General Results

Now consider a more general framework, under the following simplifying assumptions:[44]

(A1) The common value V is drawn randomly from a uniform distribution on the domain $(\underline{v}, \bar{v})$.

[44] Assumptions (A1) and (A2) were widely used by John Kagel and Dan Levin in their various experiments on common-value auctions. See for example Kagel and Levin (1986).

(A2) Before bidding, each bidder receives a private signal S_i, drawn randomly from a uniform distribution on $(V - \epsilon, V + \epsilon)$. The unknown common value determines the location of the signals' support. A lower ϵ indicates greater signal precision.

(A3) The auction is first-price, like a Dutch auction or a sealed-bid auction.[45]

As an illustration you may think of oil companies interested in the drilling rights to a particular site that is worth the same to all bidders. Each bidder obtains an estimate of the site's value from its experts and then uses this information in making a bid.

Computing the Right Expected Value

Just as in the private-values framework, each bidder has to determine the item's expected value and then strategically shade his bid, taking a bet on rival bidders' valuations. Without shading the bid, there is no chance to gain from bidding. The difference is that it is a bit more tricky to determine the right expected value, in particular since this computation should allow for the built-in adverse selection bias.

For signal values from the interval $s_i \in [v - \epsilon, v + \epsilon]$, the *expected value* of the item conditional on the signal s_i is[46]

$$E[V|S_i = s_i] = s_i. \tag{8.87}$$

This estimate is unbiased in the sense that the average estimate is equal to the site's true value.

Nevertheless, bids should not be based on this expected value. Instead, a bidder should anticipate that he would revise his estimate whenever he actually wins the auction. As in many other contexts, the clue to rational behavior is in thinking one step ahead.

In a symmetric equilibrium, a bidder wins the auction if he actually received the highest signal.[47] Therefore, when a bidder learns that he won the auction, he knows that his signal s_i was the largest received by all bidders. Using this information, he should value the item by its expected value conditional on having the highest signal, denoted by $E[V \mid S_{\max} = s_i]$, $S_{\max} := \max\{S_1, \dots, S_n\}$.

Evidently, the *updated expected value* is lower:

$$\begin{aligned} E[V \mid S_{\max} = s_i] &= s_i - \epsilon \frac{n-1}{n+1} \\ &< E[V \mid S_i = s_i]. \end{aligned} \tag{8.88}$$

Essentially, a bidder should realize that if he wins, it is likely that the signal he received was unusually high, relative to those received by rival bidders.

Notice that the adverse selection bias, measured by the difference between the two expected values, is increasing in n and ϵ. Therefore, raising the number of

45 Second-price common-value auctions are easier to analyze, but less instructive. A nice and simple proof of their equilibrium properties, based entirely on the principle of *iterated dominance*, is in Harstad and Levin (1985). The procedure proposed in this paper requires, however, that the highest signal be a *sufficient statistic* of the entire signal vector.

46 Here and elsewhere we ignore signals near corners, that is $v \notin (\underline{v}, \underline{v} + \epsilon)$ and $v \notin (\bar{v} - \epsilon, \bar{v})$.

47 This standard property follows from the monotonicity of the equilibrium bid function; in the present context, it is confirmed by the equilibrium bid function stated below.

bidders or lowering the precision of signals gives rise to a higher winner's curse, if bidders are subject to judgmental failure and base their bid on $E[V \mid S_i = s_i]$ rather than on $E[V \mid S_{\max} = s_i]$.

Equilibrium Bids

The symmetric Nash equilibrium of the common-value auction game was found by R.B. Wilson (1977b) and later generalized by Milgrom and Weber (1982) to cover auctions with a combination of private- and common-value elements.

In the present framework, the symmetric equilibrium bid function is[48]

$$b(s_i) = s_i - \epsilon + \phi(s_i), \tag{8.89}$$

where

$$\phi(s_i) := \frac{2\epsilon}{n+1} e^{-(n/2\epsilon)[s_i - (\underline{v}+\epsilon)]}. \tag{8.90}$$

The function $\phi(s_i)$ diminishes rapidly as s_i increases beyond $\underline{v} + \epsilon$. Ignoring it, the bid function is approximately equal to $b(s_i) = s_i - \epsilon$, and the expected profit of the high bidder is positive and equal to $2\epsilon/(n+1)$.

Conclusions

The main lesson to be learned from this introduction to common-value auctions is that bidders should shade their bids, for two different reasons. First, without shading bids there can be no profits in a first-price auction. Second, the auction always selects the one bidder as winner who received the most optimistic estimate of the item's value, and thus induces an adverse selection bias. Without shading the bid to allow for this bias, the winning bidder regrets his bid and falls prey to the winner's curse.

The discount associated with the first reason for shading bids will decrease with increasing number of bidders, because signal values are more congested if there are more bidders. In contrast, the discount associated with the adverse selection bias increases with the number of bidders, because the adverse selection bias becomes more severe as the number of bidders is increased.

Thus, increasing the number of bidders has two conflicting effects on equilibrium bids. On the one hand, competitive considerations require more aggressive bidding. On the other hand, allowing for the adverse selection bias requires greater discounts. In the above example with bid function (8.89), the adverse selection bias predominates, and bids are always decreasing in n. But for a number of common distribution functions such as the normal and lognormal case, the competitive effect prevails for small numbers of bidders, where the bid function is increasing in n; but eventually the adverse selection effect takes over and the equilibrium bid function decreases in n (see Wilson (1992)).

Altogether, it is clear that common-value auctions are more difficult to play and that unsophisticated bidders may be susceptible to the winner's curse. Of course,

[48] Again, we restrict the analysis to signal values from the interval $s_i \in [\underline{v} + \epsilon, \bar{v} - \epsilon]$.

the winner's curse cannot occur if bidders are rational and take proper account of the adverse selection bias. The winner's curse is strictly a matter of judgmental failure; rational bidders do not fall prey to it.

The experimental evidence demonstrates that it is often difficult to avoid the winner's curse. Even experienced subjects who are given plenty of time and opportunity to learn often fail to bid below the updated expected value. And most subjects fail to bid more conservatively when the number of bidders is increased.[49]

Similarly, many real-life decision problems are plagued by persistent winner's-curse effects. For example, in the field of book publishing, Dessauer (1981) reports that "... most of the auctioned books are not earning their advances." Capen, Clapp, and Campbell (1971) claim that the winner's curse is responsible for the low profits of oil and gas corporations on drilling rights in the Gulf of Mexico during the 1960s. And Bhagat, Shleifer, and Vishny (1990) observe that most of the major corporate takeovers that made the financial news during the 1980s have actually reduced the bidding shareholders' wealth (see also Roll (1986)).

8.7 Affiliated Values*

The assumption of independent private values is appropriate for nondurable consumption goods and services, but it is far too restrictive when it comes to the auctioning of durable goods.

As an example, consider the auctioning of a well-known painting. In order to apply the independent-private-values model, two prerequisites must be met:

1. each bidder must have his own private valuation, and
2. private valuations must be stochastically independent.

The first assumption rules out any consideration concerning a possible resale in the future, and the second assumption rules out the influence of fashion.

In this regard, the common-value model may seem more appealing, because it assumes that valuations are (perfectly) correlated. However, it completely ignores differences in bidders' preferences.

Clearly, one would like to have a more general model that combines both common- and private-value aspects and covers both basic auction models as special cases. This leads us to the *affiliated-values model* by Milgrom and Weber (1982), which is one of the most influential contributions to auction theory.

8.7.1 Private and Common Value Generalized

We now assume that bidders' valuations are influenced by their own preferences but are also dependent upon other bidders' quality assessment. Since each bidder knows only his own assessment, bidders do not exactly know their valuation of the object for sale.

[49] A comprehensive review of the experimental evidence is in Kagel (1995).

Specifically, we assume that the valuation of bidder i, V_i, is a function of all bidders' quality signals $X_1, X_2, \ldots, X_n$ and of the seller's quality signal X_0:

$$V_i := u(X_0, X). \tag{8.91}$$

The function u is nonnegative, continuous, monotone increasing, and symmetric in $\{X_j\}_{j \neq i}$; moreover, $E[V_i]$ is finite.

This formulation is sufficiently general to include both basic auction models as special cases. In particular, the private-values model is obtained by setting $V_i = X_i$, and the common-value model by setting $V_i = X_0$.

In the following we consider only symmetric equilibria. This allows us to view bidders' behavior from the perspective of one particular bidder, say bidder 1, who has observed the quality signal X_1.

We denote the order statistics of bidders' quality signals by $Y_1 \geq Y_2, \ldots, Y_{n-2} \geq Y_{n-1}$. Due to the symmetry assumption, we can thus write the valuation of bidder 1 as follows:

$$V_1 = u(X_1, Y_1, \ldots, Y_{n-1}, X_0). \tag{8.92}$$

8.7.2 Stochastic Similarity: Affiliation

As a second modification we now replace stochastic independence by affiliation. Affiliation is a statistical concept. Roughly speaking, the affiliation of random variables means that similarity of realizations is the most likely outcome. For example, if quality signals are affiliated, and bidder i has observed a high quality signal X_i, then he should believe that rival bidders have most likely also observed a high signal.

Let $f(x)$ denote the joint probability density function (pdf) of $X := (X_0, X_1, \ldots, X_n)$. Consider two realizations $x \neq x'$, and denote the componentwise maximum of (x, x') by $x \vee x'$ and the componentwise minimum by $x \wedge x'$. Quality signals are called *affiliated* if the following inequality holds for all x, x':

$$f(x \vee x') f(x \wedge x') \geq f(x) f(x'). \tag{8.93}$$

As an illustration, consider the two realizations $x := (10, 2)$ and $x' := (5, 7)$. The associated *similar* realizations are $x \vee x' = (10, 7)$ and $x \wedge x' = (5, 2)$.

Affiliation of signals (X, X_0) implies that the conditional expected values of V_1,

$$w(x, y, z) := E\left[V_1 \mid X_1 = x,\ \ Y_1 = y,\ \ X_0 = z\right], \tag{8.94}$$

$$v(x, y) := E\left[V_1 \mid X_1 = x,\ \ Y_1 = y\right], \tag{8.95}$$

are strictly monotone increasing in all variables. Note that $v(x, y)$ and $w(x, y, z)$ are the conditional expected valuations of bidder 1 after he has observed the signal realizations $X_1 = x, Y_1 = y$ and $X_1 = x,\ Y_1 = y,\ X_0 = z$, respectively.

8.7.3 Generalized Solution of the Vickrey Auction

Consider the Vickrey auction. Is truthful bidding a dominant strategy also in the present affiliated-values framework?

When the auction takes place, bidders do not exactly know their valuations. Therefore, it is unclear what "truthful bid" possibly means in this case. Does it mean that bidder 1 should bid an amount equal to his expected valuation, conditional on his own quality signal $X_1 = x$, $E[V_1 \mid X_1 = x]$? This may seem reasonable, because at the time of bidding the signal $X_1 = x$ is the best information available to bidder 1. However, beware of the selection bias. If bid functions are strictly monotone increasing, the auction is always won by the bidder who observed the most favorable quality signal. Bidders should take this additional information into account.

The wallet game from the previous section gives the clue. It suggest that bidders should bid truthfully in the particular sense of bidding *as if* the relevant rival had observed the same quality signal.

Proposition 8.15 *The Vickrey auction has a symmetric equilibrium, described by a strategy $b^*(x)$, which is a function of the own quality signal. Depending upon whether the seller keeps his own signal X_0 secret or makes it common knowledge, one has*

$$b^*(x) = v(x, x) \qquad \text{(if } X_0 \text{ is kept secret)}, \tag{8.96}$$

$$b^*(x; x_0) = w(x, x, x_0) \qquad \text{(if } X_0 \text{ is common knowledge)}. \tag{8.97}$$

Proof We sketch an intuitively accessible proof, assuming that the seller hides information concerning his own quality signal. The proof of the other case is similar and hence omitted.

In Figure 8.7 we plot the equilibrium bid function $b^*(y)$ and the conditional expected value $v(x, y) := E[V \mid X = x,\ Y_1 = y]$ for a given quality signal of the bidder $X = x$. As a working hypothesis it is assumed that b^* is strictly monotone increasing. The graphs of these functions intersect at the point $y = \hat{y}$. (Hint: for small values of y one has $v(x, y) > b^*(y)$, and for large ones $v(x, y) < b^*(y)$.)

The bidder knows his own quality signal x, but not that of his relevant rival, Y_1. Which bid is the best reply to $b^*(Y_1)$?

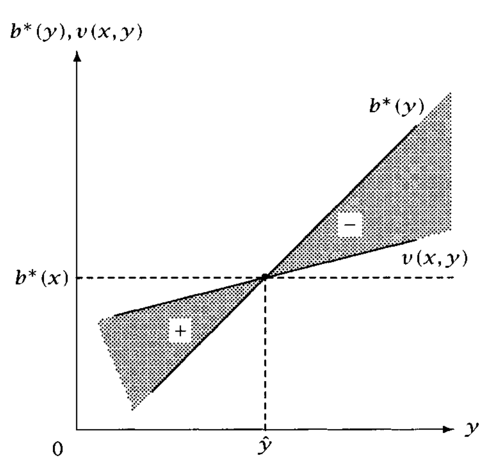

Figure 8.7. Generalized Solution of the Vickrey Auction.

The answer is simple: bidder 1 should bid the amount $b = b^*(\hat{y})$ because this way he will win if and only if his payoff $v(x, y) - b^*(y)$ is nonnegative (see Figure 8.7). Therefore, in a symmetric equilibrium one must have

$$b^*(x) = b^*(\hat{y}) = v(x, \hat{y}).$$

From the assumed strict monotonicity of b^* it follows immediately that $\hat{y} = x$, which implies the assertion $b^*(x) = v(x, x)$. And the assumed monotonicity of b^* follows from the monotonicity of $v(x, x)$. □

If a bidder succeeds with the bid $b^*(x)$, he knows for sure that the relevant rival has a lower quality signal, i.e., $Y_1 < x$. If the bidder loses, he knows for sure that $Y_1 > x$. Therefore, the implicit estimate of Y_1, implicit in the bid (8.96), is biased as follows, by the monotonicity of $v(x, y)$.

Corollary 8.1 (Biased Estimator) *Consider the equilibrium strategy of the Vickrey auction. The successful bid overestimates the quality signal of the relevant rival, Y_1, whereas the unsuccessful bid underestimates it.*

This result plays a pivotal role in the interpretation of the linkage principle, to which we turn below.

To avoid misunderstanding, note that the biased-estimator property does not reflect any deviation from optimal behavior. Just keep in mind that the winner of the Vickrey auction pays less than his own bid because the price is equal to the second highest bid.

8.7.4 Linkage Principle

The linkage principle says the following: Suppose the private quality signal of the seller is affiliated with bidders' (affiliated) private quality signals. Then a policy of always revealing the seller's private information increases the seller's expected payoff.

The seller's private information could be due to an expert assessment of a painting's authenticity or of the state of a building. The linkage principle advises the seller to make a commitment to reveal favorable as well as unfavorable information.

Proposition 8.16 *Consider the Vickrey auction with revelation (r) and without revelation (n) of the seller's quality signal X_0. Denote the associated expected values of equilibrium prices by p_r, and p_n. Then*

$$p_r = E\left[w(Y_1, Y_1, X_0) \mid \{X_1 > Y_1\}\right] \tag{8.98}$$

$$\geq E\left[v(Y_1, Y_1) \mid \{X_1 > Y_1\}\right] \tag{8.99}$$

$$= p_n. \tag{8.100}$$

Proof The proof is sketched in three steps:

1. If bidder 1 wins the auction, the equilibrium price P is equal to the highest unsuccessful bid: with revelation, $P_r = w(Y_1, Y_1, X_0)$, and without revelation, $P_n = v(Y_1, Y_1)$. Due to the assumed symmetry, these prices apply regardless of the bidder's identity.

2. Consider the price-making bid (which is the highest unsuccessful bid). If the seller keeps his information secret, it is $b^*(Y_1) = v(Y_1, Y_1)$. Due to the biased-estimator property stated in Corollary 8.1, this bid underestimates the signal of the winning bidder, which is X_1. The signals X_0 and X_1 are affiliated. Therefore, the price-making bid $v(Y_1, Y_1)$ also underestimates the distribution of X_0.
3. Now, suppose the seller is committed to reveal X_0. Then the price-making bid $w(Y_1, Y_1, X_0)$ is based on the true signal X_0. Therefore, the expected value of the equilibrium price P_r, conditioned on the true signal X_0, is higher than the expected equilibrium price P_n which is computed on the basis of a downward-biased estimated distribution of X_0. Hence, $p_r > p_n$, as asserted. □

8.7.5 Why the English Auction Benefits the Seller

There are many variants of the English auction. They differ with respect to pricing rules and the revelation of information. We exclusively consider the following variant, which is particularly favorable concerning the revelation of information: The auctioneer calls a sufficiently low initial price, which is depicted on a so-called English clock and which goes up at a preset speed. At that initial price all bidders are active, and they remain active until they irreversibly quit the auction. As the clock goes up, some bidders quit, until finally only one bidder remains active. The item is then awarded the last active bidder, who pays an amount of money equal to the price at which the last rival quit the auction.

In this auction game the strategy of a bidder is a stopping rule, which says at which price the bidder should quit, depending upon who else has quit and at which price. Let k be the number of bidders who have already quit at the prices $p_1 \leq p_2 \leq \cdots \leq p_k$. Then the strategy of a bidder with quality signal x is described by the function $b_k(x \mid p_1, \ldots, p_k)$, which indicates at which price that bidder should also quit, provided k bidders have already quit at prices $p_1, \ldots, p_k$.

As a working hypothesis, suppose the equilibrium is symmetric and the equilibrium strategy is strictly monotone increasing in x. This assures that an active bidder can deduce the underlying quality signal of those who quit, just by observing the price at which they quit.

At the beginning of the auction, when all bidders are still active, no one knows the others' quality signals. As soon as one bidder quits, the others infer that bidder's signal. The more bidders quit, the more information is revealed until, in the last round, when only two bidders remain active, all but the highest two signals are common knowledge. This built-in transmission of information allows bidders to continuously upgrade their quality assessment which, on average, leads to higher bidding, to the benefit of the seller.

Proposition 8.17 *The English auction has a symmetric equilibrium; the equilibrium strategy is a vector $b^*(x) = (b_0^*(x), \ldots, b_{n-1}^*(x))$, in which $b_k^*(x)$ is the bidders' price limit, contingent upon the prices $p_1, \ldots, p_k$ at which k bidders have* *quit the auction:*

$$b_0^*(x) = E[V_1 \mid X_1 = Y_1 = \ldots = Y_{n-1} = x], \tag{8.101}$$

$$\begin{aligned} b_k^*(x \mid p_1, \ldots, p_k) = E\,[V_1 \mid X_1 = Y_1 = \ldots &= Y_{n-k-1} = x, \\ b_{k-1}^*(Y_{n-k} \mid p_1, \ldots, p_{k-1}) &= p_k, \ldots, \\ b_0^*(Y_{n-1}) &= p_1], \quad k = 1, \ldots, n-2. \end{aligned} \tag{8.102}$$

Proof (Sketch) Suppose bidder 1 observes the signal $X_1 = x$, and all rival bidders quit exactly when the price reaches the level $p = b_0^*(y)$. Then bidder 1 knows for sure that $Y_1 = Y_2 = \cdots = Y_{n-1} = y$ wins the auction, and earns the payoff

$$E[V_1 \mid X_1 = x, \ Y_1 = Y_2 = \cdots, \ Y_{n-1} = y] - b_0^*(y).$$

Using a diagram similar to Figure 8.7, it becomes clear that (8.101) is the best reply of bidder 1 if everyone else plays the strategy (8.101).

In a similar fashion one can show that $b_1^*(x)$ is the best reply to $b_0^*(y), b_1^*(y)$, etc. Finally, the strategies are strictly monotone increasing in the own quality signal, which confirms the assumed monotonicity of all components of $b_k(x)$. □

Proposition 8.18 *The seller's expected profit is higher in the English than in the Vickrey auction.*

Proof (Sketch) In the English auction there are two phases: In the first phase the bidders with the lowest $n - 2$ quality signals reveal their signal realizations by quitting the auction at prices $p_1 \leq p_2 \leq p_{n-2}$. In the second phase the remaining two bidders play a Vickrey auction. By Proposition 8.16 it is known that revealing the seller's signal increases the seller's expected profit. Similar reasoning explains why revealing some bidders' signals raises the seller's expected profit as well. □

Milgrom and Weber (1982) generalized this result, as follows:

Proposition 8.19 *The equilibrium expected profits of the four most common auctions rank as follows:*

$$\textit{Dutch} = \textit{first-price} \leq \textit{Vickrey} \leq \textit{English}. \tag{8.103}$$

The relatively low profitability of the Dutch and first-price auctions is due to the fact that in the course of these auctions no useful information is revealed.

This result is often used to justify the use of an open, increasing auction, like the English auction. However, one can design sealed-bid auction rules that are at least as profitable as the English auction and yet less susceptible to collusion (see Perry, Wolfstetter, and Zamir (1998)).

8.7.6 Limits of the Linkage Principle

In many auctions one sells many units of identical or similar goods. And often the bidders are also interested in acquiring more than one good. Unfortunately, the theory of multiunit auctions is still fairly undeveloped. The theory of single-unit auctions can be easily generalized to multiunit auctions if each bidder demands only one unit. However, if bidders demand more than one unit, many questions are still unanswered. In particular, it is not clear whether the revenue ranking known from single-unit auctions also applies to the auctioning of many units.

Some negative results are already known. For example, we know that the linkage principle does not always apply to multiunit auctions. True, the revelation of private quality signals also raises the average unsuccessful bid and lowers the average successful bid. However, unlike in the single-unit case, it can occur that the revelation of a private quality signal turns a successful bid into an unsuccessful one, and vice versa. This upsets the linkage principle, which suggests that the English auction may be less profitable to the seller than a first-price or Dutch auction.

When practical advice is sought concerning the design of new auctions, one naturally relies on the intuition developed for the well-researched single-unit case. For example, when auction theorists were called upon to design an auction for the sale of spectrum rights for the operation of wireless telephones, they relied on the validity of the linkage principle and recommended the use of an open, increasing auction.[50] The Federal Communications Commission decided that the advantageous effect of the information revelation, which is built into an open auction, outweighs the danger of collusion, which is intrinsic to open auctions.[51]

Another problematic prerequisite of the linkage principle is the symmetry assumption. If it is common knowledge that some bidders have higher valuations than others, the linkage principle may again be upset, and the first-price or Dutch auction can be more profitable than the English auction (see Landsberger, Rubinstein, Wolfstetter, and Zamir (1998)).

Moreover, Klemperer (1998) developed several examples of auctions with *almost* common values where the seller is better off if bidders receive no information concerning other bidders' valuations. This again suggests that there are close limits to the validity of the linkage principle, and that open, increasing auctions may not always be the most profitable for the seller. To close with a positive perspective: This just shows you that auction theory is still a rich mine for further research.

8.8 Further Applications*

We close with a few remarks on the use of the auction selling mechanism in financial markets, in dealing with the natural-monopoly problem, and in oligopoly theory. Each application poses some unique problem.

8.8.1 Auctions and Oligopoly

Recall the analysis of price competition and the Bertrand paradox ("two is enough for competition") in Chapter 3 on Oligopoly and Industrial Organization. There

[50] See McMillan (1994), Milgrom (1998), and Milgrom (1997).
[51] Evidence concerning widespread collusion was reported in the *Economist,* May 17, 1997.

we elaborated on one proposed resolution of the Bertrand paradox that had to do with capacity-constrained price competition and that culminated in a defense of the Cournot model. This resolution was successful, though a bit complicated – but unfortunately not quite robust with regard to the assumed rationing rule.

A much simpler resolution of the Bertrand paradox can be found by introducing incomplete information. This explanation requires a marriage of oligopoly and auction theory, which is why we sketch it here in two examples.

Example 8.8 (Another Resolution of the Bertrand Paradox) Consider a simple Bertrand oligopoly game with inelastic demand and $n \geq 2$ identical firms. Each firm has constant unit costs c and unlimited capacity. Unlike in the standard Bertrand model, each firm knows its own cost, but not those of others. Rivals' unit costs are viewed as identical and independent random variables, described by a uniform distribution with the support $[0, 1]$.

Since the lowest price wins the market, the Bertrand game is a Dutch auction with the understanding that we are dealing here with a "buyers' auction" rather than the "seller's auction" considered before. Since costs are independent and identically distributed, we can employ the apparatus and results of the *SIPV* auction model. Strategies are price functions that map each firm's unit cost c into a unit price $p(c)$.

The unique equilibrium price strategy is characterized by the markup rule

$$p^*(c) = \frac{1}{n} + \frac{n-1}{n}c. \tag{8.104}$$

The equilibrium price is $p^*(C_{(1)})$, the equilibrium expected price[52]

$$\bar{p}(n) := E\left[p^*(C_{(1)}\right] = E\left[C_{(2)}\right] = \frac{2}{n+1}, \tag{8.105}$$

and each firm's equilibrium expected profit

$$\bar{\pi}(c, n) = \frac{(1-c)^n}{n}. \tag{8.106}$$

Evidently, more competition leads to more aggressive pricing, beginning with the monopoly price $p^*(1) = 1$ and approaching the competitive pricing rule $p = c$ as the number of oligopolists becomes very large. Similarly, the expected-price rule and profit are diminishing in n, beginning with $\bar{p}(1) = 1, \bar{\pi}(1) = 1 - c$, and with $\lim_{n\to\infty} \bar{p}(n) = 0$ and $\lim_{n\to\infty} \bar{\pi}(n) = 0$. Hence, numbers matter. Two is not enough for competition.

In order to prove these results, all we need to do is to use a transformation of random variables and then apply previous results on Dutch auctions.

For this purpose, define

$$b := 1 - p \quad \text{and} \quad v := 1 - c. \tag{8.107}$$

[52] Notice that $E[p^*(C_{(1)}] = E[C_{(2)}]$ is the familiar *revenue equivalence* that holds for all distribution functions. (If the market were second-price, each firm would set $p^*(c) \equiv c$, the lowest price would win, and trade would occur at the second lowest price; therefore, the expected price at which trade occurs would be equal to $E[C_{(2)}]$.)

This transformation maps the price competition game into a Dutch auction, where the highest bid b (the lowest p) wins. Also, $b, v \in [0, 1]$, and v is uniformly distributed. Therefore, we can apply the equilibrium bid function from Example 8.2, which was $b(v) := [(n-1)/n]v$. We conclude

$$\begin{aligned} p^* &= 1 - b \\ &= 1 - \frac{n-1}{n} v \\ &= 1 - \frac{n-1}{n}(1-c) \\ &= \frac{1}{n} + \frac{n-1}{n} c, \end{aligned} \tag{8.108}$$

as asserted.

Remark 8.3 (Why Numbers Matter) By revenue equivalence one has

$$\bar{p}(n) := E\left[p^*(C_{(1)}\right] = E\left[C_{(2)}\right]. \tag{8.109}$$

Therefore, the "two is enough for competition" property of the Bertrand paradox survives in terms of expected values. Still, numbers matter, essentially because $E[C_{(1)}]$ and $E[C_{(2)}]$ move closer together as the size n of the sample is increased. The gap between these statistics determines the aggressiveness of pricing because the price function has the form $p^*(c) = (1-\beta) + \beta c$ with

$$\beta := \frac{n-1}{n} = \frac{1 - E\left[C_{(2)}\right]}{1 - E\left[C_{(1)}\right]}. \tag{8.110}$$

Example 8.9 (Extension to Price-Elastic Demand*) Suppose demand is a decreasing function of price, as is usually assumed in oligopoly theory.[53] Specifically, assume the simplest possible inverse demand function $P(X) := 1 - X$. Otherwise, maintain the assumptions of the previous example. Then the equilibrium is characterized by the following markup rule $p^* : [0, 1] \to [0, 1]$:

$$p^*(c) = \frac{1 + nc}{n+1}. \tag{8.111}$$

Obviously, $p^*(1) = 1$ and $p^*(0) = 1/(n+1)$.

Compare this with the equilibrium markup rule in the inelastic-demand framework analyzed in the previous example, which was $p^*(c) = [1 + (n-1)c]/n$. Evidently, pricing is more aggressive if demand responds to price. Of course, if the auction were second-price, bidding would not be affected, since $p^*(c) = c$ is a dominant strategy in this case.

We conclude: If demand is a decreasing function of price, a first-price auction leads to higher expected consumer surplus. This may explain why first-price sealed bids are the common auction form in industrial and government procurement.

[53] This case was considered by Hansen, as already noted in Section 8.3.6.

This is just the beginning of a promising marriage of oligopoly and auction theory. Among the many interesting extensions, an exciting issue concerns the analysis of the repeated Bertrand game. The latter is particularly interesting because it leads to a dynamic price theory where agents successively learn about rivals' costs, which makes the information structure itself endogenous.

8.8.2 Natural-Gas and Electric Power Auctions

As a next example, recall the natural-monopoly problem discussed in Chapter 2. There we discussed regulatory mechanisms designed to minimize the cost of monopoly, when monopolization is desirable due to indivisibilities in production and associated economies of scale.

In many practical applications, it has often turned out that the properties of a natural monopoly apply only to a small part of an industry's activity. And vertical disintegration often contributed to reduce the natural-monopoly problem.

The public utility industry (gas and electric power) has traditionally been viewed as the prime example of natural monopoly. However, the presence of economies of scale in the production of energy has always been contested. An unambiguous natural monopoly exists only in the transportation and local distribution of energy. This suggests that the natural-monopoly problem in public utilities can best be handled when one disintegrates production, transportation, and local distribution of energy.

In the U.K. the electric power industry has been reshaped in recent years by the privatization of production combined with the introduction of electric power auctions. These auctions are run on a daily basis by the National Grid Company. At 10 a.m. every day, the suppliers of electric power make a bid for each of their generators to be operated on the following day. By 3 p.m., the National Grid Company has finalized a plan of action for the following day in the form of a merit order, ranked by bids from low to high. Depending upon the random demand on the following day, the spot price of electric power is then determined in such a way that demand is matched by supply, according to the merit order determined on the previous day.

Similar institutional innovations have been explored, dealing with the transportation of natural gas in long-distance pipelines and in the allocation of airport landing rights in the U.S. Interestingly, these innovations are often evaluated in laboratory experiments before they are put to a real-life test.[54]

A similarity between auctions in a regulatory setting and oligopoly with price-elastic demand is that bidders typically decide on a two-dimensional object: a price bid and a nonprice variable. For example, the nonprice bid could be quantity, as in the oligopoly application, or quality of service in a regulatory problem. Formally, the oligopoly problem with elastic demand is equivalent to a procurement problem where the social planner faces a declining social benefit function.

[54] See for example Rassenti, Smith, and Bulfin (1982) and McCabe, Rassenti, and Smith (1989).

The literature that discusses optimal mechanisms in this kind of environment includes Laffont and Tirole (1987). Che (1993) showed that the optimal mechanism can be implemented by a two-dimensional version of a first- or second-price auction that uses a *scoring rule* of the kind often seen in auctions by the U.S. Department of Defense. Typically, the optimal scoring rule gives exaggerated weight to quality.

8.8.3 Treasury-Bill Auctions

Each week the U.S. Treasury uses a discriminatory auction to sell Treasury bills (T–bills).[55] On Tuesday the Treasury announces the amount of 91-day and 182-day bills it wishes to sell on the following Monday and invites bids for specified quantities. On Thursday, the bills are issued to the successful bidders. Altogether, in fiscal year 1991 the Treasury sold over \$1.7 trillion of marketable Treasury securities (bills, notes, and bonds).

Prior to the early 1970s, the most common method of selling T-bills was that of a *subscription offering*. The Treasury fixed an interest rate on the securities to be sold and then sold them at a fixed price. A major deficiency of this method of sale was that market yields could change between the announcement and the deadline for subscriptions.

The increased market volatility in the 1970s made fixed-price offerings too risky. Subsequently, the Treasury switched to an auction technique in which the coupon rate was still preset by the Treasury, and bids were made on the basis of price. The remaining problem with this method was that presetting the coupon rate still required forecasting interest rates, with the risk that the auction price could deviate substantially from the par value of the securities.

In 1974 the Treasury switched to auctioning coupon issues on a yield basis. Thereby, bids were accepted on the basis of an annual percentage yield with the coupon rate based the weighted-average yield of accepted competitive tenders received in the auction. This freed the Treasury from having to preset the coupon rate.

Another sale method was used in several auctions of long-term bonds in the early 1970s. This was the closed, uniform-price auction method.[56] Here, the coupon rate was preset by the Treasury, and bids were accepted in terms of price. All successful bidders were awarded securities at the lowest price of accepted bids.

In the currently used auction technique, two kinds of bids can be submitted: *competitive* and *noncompetitive*. Competitive bidders are typically financial intermediaries who buy large quantities. A competitive bidder indicates the number of bills he wishes to buy and the price he is willing to pay. Multiple bids are permitted. Noncompetitive bidders are typically small or inexperienced bidders. Their bids indicate the number of bills they wish to purchase (up to \$1,000,000), with the

[55] For more details see Tucker (1985).

[56] In the financial community, a single-price, multiple-unit auction is often called "Dutch" (except by the English, who call it "American"). Be aware of the fact that in the academic literature one would never call it Dutch. Because of its second-price quality one would perhaps call it "English."

understanding that the price will be equal to the quantity-weighted average of all accepted competitive bids.

When all bids have been made, the Treasury sets aside the bills requested by noncompetitive bidders. The remainder is allocated among the competitive bidders beginning with the highest bidder, until the total quantity is placed. The price to be paid by noncompetitive bidders can then be calculated.

T-bill auctions are unique discriminatory auctions because of the distinct treatment of competitive and noncompetitive bids. Compared to the standard discriminatory auction, there is an additional element of uncertainty because competitive bidders do not know the exact amount of bills auctioned to them.

Another distinct feature is the presence of a secondary after-auction market and of pre-auction trading on a when-issued basis, which serves a price-discovering purpose where dealers typically engage in short sales.

In many countries, central banks use similar discriminatory auctions to sell short-term repurchase agreements to provide banks with short-term liquidity.[57] Several years ago, the German Bundesbank switched from a uniform-price auction to a multiple-price, discriminatory auction. The uniform-price auction had induced small banks to place very high bids because this gave them sure access to liquidity without the risk of having a significant effect on the price to be paid. Subsequently, large banks complained that this procedure put them at a competitive disadvantage, and the Bundesbank responded by switching to a discriminatory auction. However, this problem could also have been solved without changing auction procedures simply by introducing a finer grid of feasible bids (see Nautz (1997)).

In recent years, the currently used discriminatory auction procedures have become the subject of considerable public debate. Many observers have claimed that discriminatory auctions invite strategic manipulations and, perhaps paradoxically, lead to unnecessarily low revenues. Based on these observations, several prominent economists have proposed to replace the discriminatory by uniform-price auction procedures.

The recent policy debate was triggered by an inquiry into illegal bidding practices by one of the major security dealers, Salomon Brothers. Apparently, during the early 1990s, Salomon Brothers repeatedly succeeded in cornering the market by buying up to 95% of a security issue. This was in violation of U.S. regulations that do not allow a bidder to acquire more than 35% of an issue.[58] Such cornering of the market tends to be profitable because many security dealers engage in short sales between the time when an issue is announced and the time it is actually issued. This makes them vulnerable to a short squeeze.

Typically, a short squeeze develops during the when-issued period before a security is auctioned and settled. During this time, dealers already sell the soon-to-

[57] The procedures used by the Federal Reserve are described in *The Federal Reserve System: Purposes and Functions*, Board of Governors of the Federal Reserve System, Washington, D.C.

[58] Salomon Brothers circumvented the law by placing unauthorized bids in the names of customers and employees at several Treasury auctions. When these practices leaked, the Treasury security market suffered a substantial loss of confidence. For a detailed assessment of this crisis and some recommended policy changes consult the *Joint Report on the Government Securities Market* (Brady, Breeden, and Greenspan (1992)).

be-available securities and thus incur an obligation to deliver on the issue date. Of course, dealers must later cover this position, either by buying back the security at some point in the when-issued market, or in the auction, or in the post-auction secondary market, or in any combination of these. If those dealers who are short do not bid aggressively enough in the auction, they may have difficulty in covering their positions in the secondary market.

Are discriminatory auctions the best choice? Or should central banks and the Treasury go back to single-price auction procedures? This is still an exciting and important research issue. Already during the early 1960s, Friedman (1964) made a strong case in favor of a uniform-price sealed-bid auction. He asserted that this would end cornering attempts by eliminating gains from market manipulation. And, perhaps paradoxically, he also claimed that total revenue would be raised by surrendering the possibility to price-discriminate. Essentially, both claims are based on the expectation that the switch in auction rules would completely unify the primary and secondary markets and induce bidders to reveal their true willingness to pay.[59]

8.9 Bibliographic Notes

There are several good surveys of auction theory, in particular Phlips (1988), McAfee and McMillan (1987), Milgrom (1987), Milgrom (1989), Wilson (1992), and Matthews (1995b). Laffont (1997) reviews some of the empirical work on auctions. Cramton (1998) debates the pros and cons of open vs. closed (sealed-bid) auctions. And Krishna and Perry (1998) provide a unified treatment of optimal multiunit auctions and public-good problems based on a generalized Vickrey–Clarke–Groves mechanism.

Friedman and Rust (1993) review the literature on *double auctions*, which we have completely ignored here. Milgrom (1997, 1998), McMillan (1994), and Cramton (1995) review the experience with the auctioning of spectrum licenses in the U.S. And Schmidt and Schnitzer (1997) discuss the role of auctions in the privatization of public enterprises.

8.10 Appendix: Second-Order Conditions (Pseudoconcavity)

When we derived the symmetric equilibrium of SIPV auctions, we (implicitly) employed the first-order condition

$$U_b'(b, v) = \rho'(b)v - \mathcal{E}'(b) = 0, \tag{8.112}$$

which underlies the envelope theorem. But we did not verify that a sufficient second-order condition holds. Here, in this appendix, we make up for that omission.

The second-order condition that we employ is not your usual concavity or quasi-concavity condition. Instead, we show that $U(b, v)$ is increasing in b for $b < b^*(v)$,

[59] For a discussion of Friedman's ideas, see Goldstein (1962) and Bikchandani and Huang (1989, 1998).

and decreasing for $b > b^*(v)$. Naturally, this implies that bidders' payoff is maximized at $b = b^*(v)$. In the optimization literature this second-order condition is known under the name of *pseudoconcavity*.[60]

Differentiate $U_b'(b, v)$ with respect to v, and one has

$$U_{bv}''(b, v) = \rho'(b) > 0. \tag{8.113}$$

Now, suppose $b < b^*(v)$. Let $\hat{v}$ be the valuation that would lead to the bid b if strategy b^* were played, that is, $b^*(\hat{v}) = b$. By the strict monotonicity of b^* it follows that one must have $v > \hat{v}$. Therefore, by (8.113), for all $b < b^*(v)$,

$$U_b'(b, v) \geq U_b'(b, \hat{v}) \equiv U_b'(b^*(\hat{v}), \hat{v}) \equiv 0. \tag{8.114}$$

In words, $U(b, v)$ is increasing in b for all $b < b^*(v)$.

Similarly one can show that $U(b, v)$ is decreasing in b for all $b > b^*(v)$. Therefore, $b^*(v)$ is a global maximizer, and hence $b^*(v)$ is an equilibrium.

[60] A differentiable function $f : \mathbb{R}^n \to \mathbb{R}$ is called *pseudoconcave* if for all $x, y \in \mathbb{R}^n$, $(y - x)\nabla f(x) \leq 0 \Longrightarrow f(y) \leq f(x)$. Pseudoconcavity is a sufficient second-order condition, because if f is pseudoconcave and the gradient vector is $\nabla f(x^*) = 0$, then x^* maximizes f. Note that if $n = 1$, as in the present context, the graph of a pseudoconcave function is single-peaked.

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