

Microeconomic Analysis

Third Edition

HAL R. VARIAN

NORTON



INTERNATIONAL
STUDENT EDITION

To my parents

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CHAPTER 14

MONOPOLY

The word **monopoly** originally meant the right of exclusive sale. It has come to be used to describe any situation in which some firm or small group of firms has the exclusive control of a product in a given market. The difficulty with this definition comes in defining what one means by a “given market.” There are many firms in the soft-drink market, but only a few firms in the cola market.

The relevant feature of a monopolist from the viewpoint of economic analysis is that a monopolist has market power in the sense that the amount of output that it is able to sell responds continuously as a function of the price it charges. This is to be contrasted to the case of a competitive firm whose sales drop to zero if it charges a price higher than the prevailing market price. A competitive firm is a *price-taker*; a monopoly is a *price-maker*.

The monopolist faces two sorts of constraints when it chooses its price and output levels. First, it faces the standard technological constraints of the sort described earlier—there are only certain patterns of inputs and outputs that are technologically feasible. We will find it convenient to summarize the technological constraints by the use of the cost function,

$c(y)$. (We omit the factor prices as an argument in the cost function since we will assume that they are fixed.)

The second set of constraints that the monopolist faces is that presented by the consumers' behavior. The consumers are willing to purchase different amounts of the good at different prices, and we summarize this relationship using the demand function, $D(p)$.

The monopolist's profit maximization problem can be written as

$$\begin{aligned} & \max_{p,y} py - c(y) \\ & \text{such that } D(p) \geq y \end{aligned}$$

In most cases of interest, the monopolist will want to produce the amount that the consumers demand, so the constraint can be written as the equality $y = D(p)$. Substituting for y in the objective function, we have the problem

$$\max_p pD(p) - c(D(p)).$$

Although this is perhaps the most natural way to pose the monopolist's maximization problem, it turns out to be more convenient in most situations to use the *inverse* rather than the direct demand function.

Let $p(y)$ be the inverse demand function—the price that must be charged to sell y units of output. Then the revenue that the monopolist can expect to receive if it produces y units of output is $r(y) = p(y)y$. We can pose the monopolist's maximization problem as

$$\max_y p(y)y - c(y).$$

The first- and second-order conditions for this problem are

$$p(y) + p'(y)y = c'(y) \quad (14.1)$$

$$2p'(y) + p''(y)y - c''(y) \leq 0. \quad (14.2)$$

The first-order condition says that at the profit-maximizing choice of output marginal revenue must equal marginal cost. Let us consider this condition more closely. When the monopolist considers selling dy units more output, it has to take into account two effects. First, its revenues increase by $p dy$ because it sells more output at the current price. But second, in order to sell this additional output it must reduce its price by $dp = \frac{dp}{dy} dy$, and this lower price applies to all the units y it is selling. The additional revenue from selling the additional output is therefore given by

$$p dy + dp y = \left[p + \frac{dp}{dy} y \right] dy,$$

and it is this quantity that must be balanced against marginal cost.

The second-order condition requires that the derivative of marginal revenue must be less than the derivative of marginal cost; i.e., the marginal revenue curve crosses the marginal cost curve from above.

The first-order condition can be rearranged to take the form

$$r'(y) = p(y) \left[1 + \frac{dp}{dy} \frac{y}{p} \right] = c'(y),$$

or

$$p(y) \left[1 + \frac{1}{\epsilon(y)} \right] = c'(y), \quad (14.3)$$

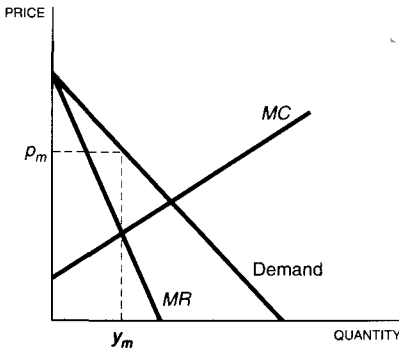
where

$$\epsilon(y) = \frac{p}{y} \frac{dy}{dp}$$

is the (price) **elasticity of demand** facing the monopolist. Note that the elasticity will be a negative number as long as the consumers' demand curve has a negative slope, which is certainly the standard case.

It follows from the first-order condition that at the optimal level of output the elasticity of demand must be greater than 1 in absolute value. If this were not the case, marginal revenue would be negative and hence could not be equal to the nonnegative marginal cost.

The optimal output of the monopolist is represented graphically in Figure 14.1. The marginal revenue curve is given by $r'(y) = p(y) + p'(y)y$. Since $p'(y) < 0$ by assumption, the marginal revenue curve lies beneath the inverse demand curve.



Determination of the monopoly output. The monopolist produces where marginal revenue equals marginal cost.

Figure 14.1

When $y = 0$ the marginal revenue from selling an extra unit of output is just the price $p(0)$. However, when $y > 0$, the marginal revenue from

selling an extra unit of output must be less than the price since the only way to sell the additional output is to reduce the price, and this reduction in the price will affect the revenue received from all the inframarginal units sold.

The optimal level of output of the monopolist is where the marginal revenue curve crosses the marginal cost curve. In order to satisfy the second-order condition, the MR curve must cross the MC curve from above. We will typically assume that there is a unique profit-maximizing level of output. Given the level of output, say y^* , the price charged will be given by $p(y^*)$.

14.1 Special cases

There are two special cases for monopoly behavior that are worth mentioning. The first is that of linear demand. If the inverse demand curve is of the form $p(y) = a - by$, then the revenue function will be of the form $r(y) = ay - by^2$ and the marginal revenue takes the form $r'(y) = a - 2by$. Hence, the marginal revenue curve is twice as steep as the demand curve. If firm exhibits constant marginal costs of the form $c(y) = cy$, we can solve the marginal revenue equals marginal cost equations to determine the monopoly price and output directly:

$$\begin{aligned}y^* &= \frac{a - c}{2b} \\p^* &= \frac{a + c}{2}.\end{aligned}$$

The other case of interest is the constant elasticity demand function, $y = Ap^{-b}$. As we saw earlier, the elasticity of demand is constant and given by $\epsilon(y) = -b$. In this case we can apply (14.3) and write

$$p(y) = \frac{c}{1 - 1/b}.$$

Hence, for the constant elasticity demand function, price is a constant markup over marginal cost, with the amount of the markup depending on the elasticity of demand.

14.2 Comparative statics

It is often of interest to determine how the monopolist's output and price change as its costs change. Suppose for simplicity that the marginal costs are constant. Then the profit maximization problem is

$$\max_y p(y)y - cy,$$

and the first-order condition is

$$p(y) + p'(y)y - c = 0.$$

We know from the standard comparative statics calculation that the sign of dy/dc is the same as the sign of the derivative of the first-order condition with respect to c . This is easily seen to be negative, so we conclude that a profit-maximizing monopolist will always reduce its output when its marginal costs increase.

It is more interesting to calculate the effect of a cost change on price. We know from the chain rule that

$$\frac{dp}{dc} = \frac{dp}{dy} \frac{dy}{dc}.$$

It is clear from this expression that $dp/dc > 0$, but it is often useful to know the magnitude of dp/dc .

The standard comparative statics calculation tells us that

$$\frac{dy}{dc} = - \frac{\partial^2 \pi / \partial y \partial c}{\partial^2 \pi / \partial y^2}.$$

Taking the appropriate second derivatives of the profit function, we have

$$\frac{dy}{dc} = \frac{1}{2p'(y) + yp''(y)}.$$

It follows that

$$\frac{dp}{dc} = \frac{p'(y)}{2p'(y) + yp''(y)}.$$

This can also be written as

$$\frac{dp}{dc} = \frac{1}{2 + yp''(y)/p'(y)}.$$

From this expression, it is easy to see what happens in the special cases mentioned above. If demand is linear, then $p''(y) = 0$ and $dp/dc = 1/2$. If the demand function exhibits constant elasticity of ϵ , then $dp/dc = \epsilon/(1+\epsilon)$. In the case of a linear demand curve, half of a cost increase is passed along in the form of increased prices. In the case of a constant elasticity demand, prices increase by *more* than the increase in costs—the more inelastic the demand, the more of the cost increase gets passed along.

14.3 Welfare and output

We have seen in Chapter 13 that under certain conditions the level of output at which price equals marginal cost is Pareto efficient. Since the marginal revenue curve always lies under the inverse demand curve, it is clear that a monopoly must produce a level of output which is less than this Pareto efficient amount. In this section we will examine this inefficiency of monopoly in a bit more detail.

For simplicity, let us consider an economy with one consumer, possessing a quasilinear utility function $u(x) + y$. As we've seen in Chapter 13, the inverse demand function for this form of utility function is given by $p(x) = u'(x)$. Let $c(x)$ denote the amount of the y-good necessary to produce x units of the x-good. Then a sensible social objective is to choose x to maximize utility:

$$W(x) = u(x) - c(x).$$

This implies that the socially optimal level of output, x_o , is given by

$$u'(x_o) = p(x_o) = c'(x_o).$$

On the other hand, the level of monopoly output satisfies the condition

$$p(x_m) + p'(x_m)x_m = c'(x_m).$$

Hence, the derivative of the welfare function evaluated at the monopoly level of output is given by

$$W'(x_m) = u'(x_m) - c'(x_m) = -p'(x_m)x_m = -u''(x_m)x_m > 0.$$

It follows from the concavity of $u(x)$ that increasing output will increase utility.

We could make the same argument slightly differently. We can also write the social objective function as consumer's surplus plus profits:

$$W(x) = [u(x) - p(x)x] + [p(x)x - c(x)].$$

The derivative of profits with respect to x is zero at the monopoly output, since the monopolist chooses the level of output that maximizes profits. The derivative of consumer's surplus at x_m is given by

$$u'(x_m) - p(x_m) - p'(x_m)x_m = -p'(x_m)x_m,$$

which is certainly positive.

14.4 Quality choice

Monopolies choose not only output levels, but also other dimensions of the products they produce. Consider, for example, product quality. Let us suppose that we can denote product quality by some numerical level q . We suppose that both utility and costs depend on quality and take the social objective function to be

$$W(x, q) = u(x, q) - c(x, q).$$

(As usual, we assume quasilinear utility for a simple analysis.) We assume that quality is a good, so that $\partial u / \partial q > 0$ and that it is costly to produce, so that $\partial c / \partial q > 0$.

The monopolist maximizes profits:

$$\max_{x, q} p(x, q)x - c(x, q).$$

The first-order conditions for this problem are

$$\begin{aligned} p(x_m, q_m) + \frac{\partial p(x_m, q_m)}{\partial x} x_m &= \frac{\partial c(x_m, q_m)}{\partial x} \\ \frac{\partial p(x_m, q_m)}{\partial q} x_m &= \frac{\partial c(x_m, q_m)}{\partial q}. \end{aligned}$$

Let us calculate the derivative of the welfare function at (x_m, q_m) . We have

$$\begin{aligned} \frac{\partial W(x_m, q_m)}{\partial x} &= \frac{\partial u(x_m, q_m)}{\partial x} - \frac{\partial c(x_m, q_m)}{\partial x} \\ \frac{\partial W(x_m, q_m)}{\partial q} &= \frac{\partial u(x_m, q_m)}{\partial q} - \frac{\partial c(x_m, q_m)}{\partial q}. \end{aligned}$$

Upon substituting from the first-order conditions, we find

$$\frac{\partial W(x_m, q_m)}{\partial x} = -\frac{\partial p(x_m, q_m)}{\partial x} x_m > 0 \quad (14.4)$$

$$\frac{\partial W(x_m, q_m)}{\partial q} = \frac{\partial u(x_m, q_m)}{\partial q} - \frac{\partial p(x_m, q_m)}{\partial q} x_m. \quad (14.5)$$

The first equation tells us that, holding quality fixed, the monopolist produces too little output, relative to the social optimum. The second equation is not quite so easy to interpret. Since $\partial p / \partial q$ equals the marginal cost of producing more quality, it must be positive, so the derivative of welfare with respect to quality is the difference between two positive numbers and is, on the face of it, ambiguous.

The question is, can we find any plausible conditions on demand behavior that will sign the expression? This is a case where it is much easier to see the answer if we write the social objective function as consumer's surplus plus profits rather than utility minus costs. The social objective function takes the form

$$\begin{aligned} W(x, q) &= [u(x, q) - p(x, q)x] + [p(x, q)x - c(x, q)] \\ &= \text{consumer's surplus} + \text{profits.} \end{aligned}$$

Now differentiate this definition with respect to x and q and evaluate the derivative at the monopolist's profit-maximizing level of output. Since the monopolist is maximizing profits, the derivatives of monopoly profits with respect to output and quality must vanish, indicating that the derivative of welfare with respect to quantity and quality is precisely the derivative of consumer's surplus with respect to quantity and quality.

The derivative of consumer's surplus with respect to quantity is always positive, which is just another way of saying that the monopolist produces too little output. The derivative of consumer's surplus with respect to quality is ambiguous—it may be positive or it may be negative. Its sign depends on the sign of $\partial^2 p(x, q)/\partial x \partial q$.

To see this, consider Figure 14.2. When quality increases, the demand curve shifts up and (possibly) tilts one way or the other. Decompose this movement into a *parallel* shift up and a pivot, as indicated. Consumer's surplus is unaffected by the parallel shift, so the total change simply depends on whether the inverse demand curve becomes flatter or steeper. If the slope of the inverse demand curve gets flatter consumer's surplus goes down and vice-versa.¹

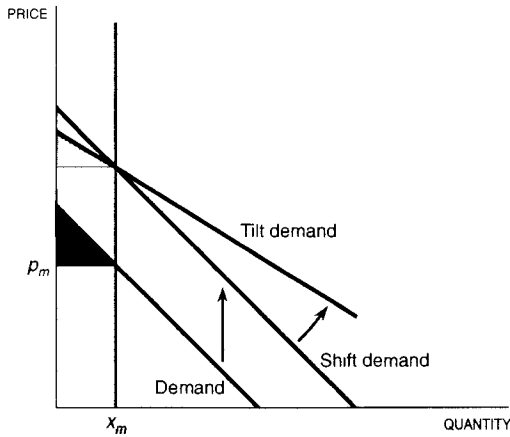
Another way to interpret equation (14.5) is based on consideration of the reservation price model. Think of $p(x, q)$ as measuring the reservation price of consumer x , so that $u(x, q)$ is just the sum of the reservation prices. In this interpretation, $u(x, q)/x$ is the average willingness to pay and $p(x, q)$ is the marginal willingness to pay. We can rewrite (14.5) as

$$\frac{1}{x_m} \frac{\partial W(x_m, q_m)}{\partial q} = \frac{\partial}{\partial q} \left[\frac{u(x_m, q_m)}{x_m} - p(x_m, q_m) \right].$$

The derivative of welfare with respect to q is now seen to be proportional to the derivative of the *average* willingness to pay for the quality change minus the derivative of the *marginal* willingness to pay for the quality change.

Social welfare depends on the sum of the consumers' utility or willingness to pay; but the monopolist only cares about the willingness to pay of the marginal individual. If these two values are different, the monopolist's quality choice will not be optimal from the social viewpoint.

¹ Note that the slope of the demand curve is negative; to say the slope gets flatter means that it gets closer to zero.



Effect on consumer's surplus of a change in quality.
When the demand curve shifts up and tilts, the effect on consumer's surplus depends only on the direction of the tilt.

Figure 14.2

14.5 Price discrimination

Loosely speaking, price discrimination involves selling different units of the same good at different prices, either to the same or different consumers. Price discrimination arises naturally in the study of monopoly since we have seen that a monopolist will typically desire to sell additional output if it can find a way to do so without lowering the price on the units it is currently selling.

In order for price discrimination to be a viable strategy for the firm, it must have the ability to sort consumers and to prevent resale. Preventing resale is generally not a severe problem, and most of the difficulties associated with price discrimination are concerned with sorting the consumers. The easiest case is where the firm can explicitly sort consumers with respect to some exogenous category such as age. A more complex analysis is necessary when the firm must price discriminate on the basis of some *endogenous* category such as the amount of purchase or the time of purchase. In this case the monopolist faces the problem of structuring its pricing so that consumers “self select” into appropriate categories.

The traditional classification of the forms of price discrimination is due to Pigou (1920).

First-degree price discrimination involves the seller charging a different price for each unit of the good in such a way that the price charged for each unit is equal to the maximum willingness-to-pay for that unit. This is also known as **perfect price discrimination**

Second-degree price discrimination occurs when prices differ depending on the number of units of the good bought, but not across consumers. This phenomenon is also known as **nonlinear pricing**. Each consumer faces the same price schedule, but the schedule involves different prices for different amounts of the good purchased. Quantity discounts or premia are the obvious examples.

Third-degree price discrimination means that different purchasers are charged different prices, but each purchaser pays a constant amount for each unit of the good bought. This is perhaps the most common form of price discrimination; examples are student discounts, or charging different prices on different days of the week.

We will investigate these three forms of price discrimination in the context of a very simple model. Suppose that there are two potential consumers with utility functions $u_i(x) + y$, for $i = 1, 2$. For simplicity, normalize utility so that $u_i(0) = 0$. Consumer i 's maximum willingness-to-pay for some consumption level x will be denoted by $r_i(x)$. It is the solution to the equation

$$u_i(0) + y = u_i(x) - r_i(x) + y.$$

The left-hand side of the equation gives the utility from zero consumption of the good, and the right-hand side gives the utility from consuming x units and paying a price $r_i(x)$. By virtue of our normalization, $r_i(x) \equiv u_i(x)$.

Another useful function associated with the utility function is the marginal willingness-to-pay function, i.e., the (inverse) demand function. This function measures what the per-unit price would have to be to induce the consumer to demand x units of the consumption good. If the consumer faces a per-unit price p and chooses the optimal level of consumption, he or she must solve the utility maximization problem

$$\begin{aligned} \max_{x,y} \quad & u_i(x) + y \\ \text{such that} \quad & px + y = m. \end{aligned}$$

As we have seen several times, the first-order condition for this problem is

$$p = u'_i(x). \quad (14.6)$$

Hence, the inverse demand function is given explicitly by (14.6): the price necessary to induce consumer i to choose consumption level x is $p = p_i(x) = u'_i(x)$.

We will suppose that the maximum willingness-to-pay for the good by consumer 2 always exceeds the maximum willingness-to-pay by consumer 1; i.e., that

$$u_2(x) > u_1(x) \text{ for all } x. \quad (14.7)$$

We will also generally suppose that the *marginal* willingness-to-pay for the good by consumer 2 exceeds the marginal willingness-to-pay by consumer 1; i.e., that

$$u'_2(x) > u'_1(x) \text{ for all } x. \quad (14.8)$$

Thus it is natural to refer to consumer 2 as the **high demand** consumer and consumer 1 as the **low demand** consumer.

We will suppose that there is a single seller of the good in question who can produce it at a constant marginal cost of c per unit. Thus the cost function of the monopolist is $c(x) = cx$.

14.6 First-degree price discrimination

Suppose for the moment that there is only one agent, so that we can drop the subscript distinguishing the agents. A monopolist wants to offer the agent some price and output combination (r^*, x^*) that yields the maximum profits for the monopolist. The price r^* is a take-it-or-leave-it price—the consumer can either pay r^* and consume x^* , or consume zero units of the good.

The profit maximization problem of the monopolist is

$$\begin{aligned} & \max_{r, x} r - cx \\ & \text{such that } u(x) \geq r. \end{aligned}$$

The constraint simply indicates that the consumer must get nonnegative surplus from his consumption of the x -good. Since the monopolist wants r to be as large as possible, this constraint will be satisfied as an equality.

Substituting from the constraint and differentiating, we find the first-order condition determining the optimal level of production to be

$$u'(x^*) = c. \quad (14.9)$$

Given this level of production, the take-it-or-leave-it price is

$$r^* = u(x^*).$$

There are several points worth noting about this solution. First, the monopolist will choose to produce a Pareto efficient level of output—a level of output where the marginal willingness-to-pay equals marginal cost. However, the producer will also manage to capture all the benefits from this efficient level of production—it will achieve the maximum possible profits, while the consumer is indifferent to consuming the product or not.

Second, the monopolist in this market produces the same level of output as would a competitive industry. A competitive industry will produce where

price equals marginal cost and supply equals demand. Together, these two conditions imply that $p(x) = c$, which is precisely the equation (14.9) coupled with the definition of the inverse demand function in (14.6). Of course, the gains from trade are divided much differently in the competitive equilibrium. In this case, the consumer gets utility $u(x^*) - cx^*$ and the firm gets zero profits.

Third, the same outcome can be achieved if the monopolist sells each unit of output to the consumer at a different price. Suppose, for example, that the firm breaks up the output into n pieces of size Δx , so that $x = n\Delta x$. Then the willingness-to-pay for the first unit of consumption will be given by

$$u(0) + m = u(\Delta x) + m - p_1,$$

or

$$u(0) = u(\Delta x) - p_1.$$

Similarly, the marginal willingness-to-pay for the second unit of consumption is

$$u(\Delta x) = u(2\Delta x) - p_2.$$

Proceeding this way up to the n units, we have the sequence of equations,

$$\begin{aligned} u(0) &= u(\Delta x) - p_1 \\ u(\Delta x) &= u(2\Delta x) - p_2 \\ &\vdots \\ u((n-1)\Delta x) &= u(x) - p_n. \end{aligned}$$

Adding up these n equations and using the normalization that $u(0) = 0$, we have $\sum_{i=1}^n p_i = u(x)$. That is the sum of the marginal willingnesses-to-pay must equal the total willingness-to-pay. So it doesn't matter how the firm price discriminates: making a single take-it-or-leave-it offer, or selling each unit of the good at the marginal willingness-to-pay for that unit.

14.7 Second-degree price discrimination

Second-degree price discrimination is also known as **nonlinear pricing**. This involves such practices as quantity discounts, where the revenue a firm collects is a nonlinear function of the amount purchased. In this section we will analyze a simple problem of this type.

Recall the notation introduced earlier. There are two consumers with utility functions $u_1(x_1) + y_1$ and $u_2(x_2) + y_2$, where we assume that $u_2(x) > u_1(x)$ and $u'_2(x) > u'_1(x)$. We refer to consumer 2 as the *high-demand* consumer and consumer 1 as the *low-demand* consumer. The assumption that the consumer with the larger total willingness-to-pay also has the larger

marginal willingness-to-pay is sometimes known as the **single crossing property** since it implies that any two indifference curves for the agents can intersect at most once.

Suppose that the monopolist chooses some (nonlinear) function $p(x)$ that indicates how much it will charge if x units are demanded. Suppose that consumer i demands x_i units and spends $r_i = p(x_i)x_i$ dollars. From the viewpoint of both the consumer and the monopolist all that is relevant is that the consumer spends r_i dollars and receives x_i units of output. Hence, the choice of the function $p(x)$ reduces to the choice of (r_i, x_i) . Consumer 1 will choose (r_1, x_1) and consumer 2 will choose (r_2, x_2) .

The constraints facing the monopolist are as follows. First, each consumer must want to consume the amount x_i and be willing to pay the price r_i :

$$u_1(x_1) - r_1 \geq 0$$

$$u_2(x_2) - r_2 \geq 0.$$

This simply says that each consumer must do at least as well consuming the x -good as not consuming it. Second, each consumer must prefer his consumption to the consumption of the other consumer.

$$u_1(x_1) - r_1 \geq u_1(x_2) - r_2$$

$$u_2(x_2) - r_2 \geq u_2(x_1) - r_1.$$

These are the so-called **self-selection constraints**. If the plan (x_1, x_2) is to be feasible in the sense that it will be voluntarily chosen by the consumers, then each consumer must prefer consuming the bundle intended for him as compared to consuming the other person's bundle.

Rearrange the inequalities in the above paragraph as

$$r_1 \leq u_1(x_1) \tag{14.10}$$

$$r_1 \leq u_1(x_1) - u_1(x_2) + r_2 \tag{14.11}$$

$$r_2 \leq u_2(x_2) \tag{14.12}$$

$$r_2 \leq u_2(x_2) - u_2(x_1) + r_1. \tag{14.13}$$

Of course, the monopolist wants to choose r_1 and r_2 to be as large as possible. It follows that in general one of the first two inequalities will be binding and one of the second two inequalities will be binding. It turns out that the assumptions that $u_2(x) > u_1(x)$ and $u'_2(x) > u'_1(x)$ are sufficient to determine which constraints will bind, as we now demonstrate.

To begin with, suppose that (14.12) is binding. Then (14.13) implies that

$$r_2 \leq r_2 - u_2(x_1) + r_1,$$

or

$$u_2(x_1) \leq r_1.$$

Using (14.7) we can write

$$u_1(x_1) < u_2(x_1) \leq r_1,$$

which contradicts (14.10). It follows that (14.12) is not binding and that (14.13) is binding, a fact which we note for future use:

$$r_2 = u_2(x_2) - u_2(x_1) + r_1. \quad (14.14)$$

Now consider (14.10) and (14.11). If (14.11) were binding, we would have

$$r_1 = u_1(x_1) - u_1(x_2) + r_2.$$

Substitute from (14.14) to find

$$r_1 = u_1(x_1) - u_1(x_2) + u_2(x_2) - u_2(x_1) + r_1,$$

which implies

$$u_2(x_2) - u_2(x_1) = u_1(x_2) - u_1(x_1).$$

We can rewrite this expression as

$$\int_{x_1}^{x_2} u'_1(t) dt = \int_{x_1}^{x_2} u'_2(t) dt.$$

However, this violates the assumption that $u'_2(x) > u'_1(x)$. It follows that (14.11) is not binding and that (14.10) is binding, so

$$r_1 = u_1(x_1). \quad (14.15)$$

Equations (14.14) and (14.15) imply that the low-demand consumer will be charged his maximum willingness-to-pay, and the high-demand consumer will be charged the highest price that will just induce him to consume x_2 rather than x_1 .

The profit function of the monopolist is

$$\pi = [r_1 - cx_1] + [r_2 - cx_2],$$

which upon substitution for r_1 and r_2 becomes

$$\pi = [u_1(x_1) - cx_1] + [u_2(x_2) - u_2(x_1) + u_1(x_1) - cx_2].$$

This expression is to be maximized with respect to x_1 and x_2 . Differentiating, we have

$$u'_1(x_1) - c + u'_1(x_1) - u'_2(x_1) = 0 \quad (14.16)$$

$$u'_2(x_2) - c = 0. \quad (14.17)$$

Equation (14.16) can be rearranged to give

$$u'_1(x_1) = c + [u'_2(x_1) - u'_1(x_1)] > c, \quad (14.18)$$

which implies that the low-demand consumer has a (marginal) value for the good that exceeds marginal cost. Hence he consumes an inefficiently small amount of the good. Equation (14.17) says that at the optimal nonlinear prices, the high-demand consumer has a marginal willingness-to-pay which is equal to marginal cost. Thus he consumes the socially correct amount.

Note that if the single-crossing property were not satisfied, then the bracketed term in (14.18) would be negative and the low-demand consumer would consume a *larger* amount than he would at the efficient point. This can happen, but it is admittedly rather peculiar.

The result that the consumer with the highest demand pays marginal cost is very general. If the consumer with the highest demand pays a price in excess of marginal cost, the monopolist could lower the price charged to the largest consumer by a small amount, inducing him to buy more. Since price still exceeds marginal cost, the monopolist would make a profit on these sales. Furthermore, such a policy wouldn't affect the monopolist's profits from any other consumers, since they are all optimized at lower values of consumption.

EXAMPLE: A graphical treatment

The price discrimination problem with self-selection can also be treated graphically. Consider Figure 14.3 which depicts the demand curves of the two consumers; for simplicity we assume zero marginal cost. Figure 14.3A depicts the price discrimination if there is no self-selection problem. The firm would simply sell x_h^o to the high-demand consumer and x_l^o to the low-demand consumer at prices that are equal to their respective consumer's surpluses—i.e., the areas under their respective demand curves. Thus the high-demand consumer pays $A + B + C$ to consume x_h^o and the low-demand consumer pays A to consume x_l^o .

However, this policy violates the self-selection constraint. The high-demand consumer prefers the low-demand consumer's bundle, since by choosing it he receives a net surplus equal to the area B . In order to satisfy the self-selection constraint, the monopolist must offer x_h^o at a price equal to $A + C$, which leaves the high-demand consumer a surplus equal to B no matter which bundle he chooses.

This policy is feasible, but is it optimal? The answer is no: by offering the low-demand consumer a slightly smaller bundle, the monopolist loses the profits indicated by the black triangle in Figure 14.3B, and gains the profits indicated by the shaded trapezoid. Reducing the amount offered to the low-demand consumer has no first-order effect on profits since the marginal

willingness-to-pay equals zero at x_l^0 . However, it increases profits non-marginally since the high-demand consumer's willingness-to-pay is *larger* than zero at this point.

At the profit-maximizing level of consumption for the low-demand consumer, x_l^m in Figure 14.3C, the marginal *decrease* in profits collected from the low-demand consumer from a further reduction, p_1 , just equals the marginal *increase* in profits collected from the high-demand consumer, $p_2 - p_1$. (Note that this also follows from equation (14.18).) The final solution has the low-demand consumer consuming at x_l^m and paying A , thereby receiving zero surplus from his purchase. The high-demand consumer consumes at x_h^0 , the socially correct amount, and pays $A + C + D$ for this bundle, leaving him with positive surplus in the amount B .

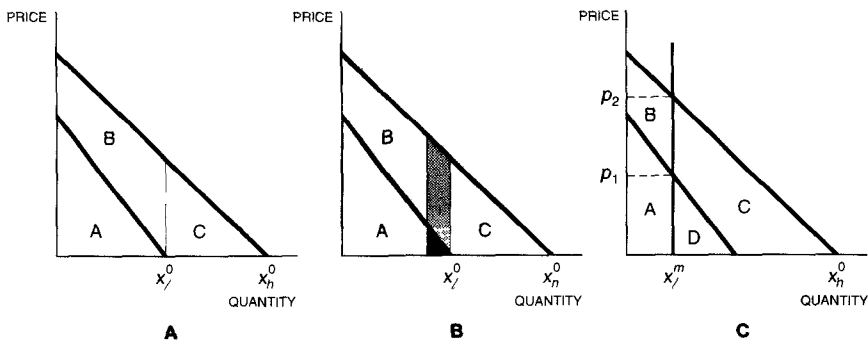


Figure 14.3

Second-degree price discrimination. Panel A depicts the solution if self-selection is not a problem. Panel B shows that reducing the bundle of the low-demand consumer will increase profits, and panel C shows the profit-maximizing level of output for the low-demand consumer.

14.8 Third-degree price discrimination

Third-degree price discrimination occurs when consumers are charged different prices, but each consumer faces a constant price for all units of output purchased. This is probably the most common form of price discrimination.

The textbook case is where there are two separate markets, where the firm can easily enforce the division. An example would be discrimination by age, such as youth discounts at the movies. If we let $p_i(x_i)$ be the inverse demand function for group i , and suppose that there are two groups, then the monopolist's profit maximization problem is

$$\max_{x_1, x_2} p_1(x_1)x_1 + p_2(x_2)x_2 - cx_1 - cx_2.$$

The first-order conditions for this problem are

$$\begin{aligned}p_1(x_1) + p'_1(x_1)x_1 &= c \\p_2(x_2) + p'_2(x_2)x_2 &= c.\end{aligned}$$

Let ϵ_i be the elasticity of demand in market i , we can write these expressions as

$$\begin{aligned}p_1(x_1) \left[1 - \frac{1}{|\epsilon_1|} \right] &= c \\p_2(x_2) \left[1 - \frac{1}{|\epsilon_2|} \right] &= c.\end{aligned}$$

It follows that $p_1(x_1) > p_2(x_2)$ if and only if $|\epsilon_1| < |\epsilon_2|$. Hence, the market with the more elastic demand—the market that is more price sensitive—is charged the lower price.

Suppose now that the monopolist is unable to separate the markets as cleanly as assumed, so that the price charged in one market influences the demand in another market. For example, consider a theater that has a bargain night on Monday; the lower price on Monday would presumably influence demand on Tuesday to some degree.

In this case the profit maximization problem of the firm is

$$\max_{x_1, x_2} p_1(x_1, x_2)x_1 + p_2(x_1, x_2)x_2 - cx_1 - cx_2,$$

and the first-order conditions become

$$\begin{aligned}p_1 + \frac{\partial p_1}{\partial x_1}x_1 + \frac{\partial p_2}{\partial x_1}x_2 &= c \\p_2 + \frac{\partial p_2}{\partial x_2}x_2 + \frac{\partial p_1}{\partial x_2}x_1 &= c.\end{aligned}$$

We can rearrange these conditions to give

$$\begin{aligned}p_1 \left[1 - \frac{1}{|\epsilon_1|} \right] + \frac{\partial p_2}{\partial x_1}x_2 &= c \\p_2 \left[1 - \frac{1}{|\epsilon_2|} \right] + \frac{\partial p_1}{\partial x_2}x_1 &= c.\end{aligned}$$

Since we are assuming quasilinear utility, it follows that $\partial p_1/\partial x_2 = \partial p_2/\partial x_1$; i.e., the cross-price effects are symmetric. Subtracting the second equation from the first and rearranging, we have

$$p_1 \left[1 - \frac{1}{|\epsilon_1|} \right] - p_2 \left[1 - \frac{1}{|\epsilon_2|} \right] = [x_1 - x_2] \frac{\partial p_2}{\partial x_1}.$$

It is natural to suppose that the two goods are substitutes—after all they are the same good being sold to different groups—so that $\partial p_2/\partial x_1 > 0$.

Without loss of generality, assume that $x_1 > x_2$, which, by the equation immediately above, implies that

$$p_1 \left[1 - \frac{1}{|\epsilon_1|} \right] - p_2 \left[1 - \frac{1}{|\epsilon_2|} \right] > 0.$$

Rearranging, we have

$$\frac{p_1}{p_2} > \frac{1 - 1/|\epsilon_2|}{1 - 1/|\epsilon_1|}.$$

It follows from this expression that if $|\epsilon_2| > |\epsilon_1|$ we must have $p_1 > p_2$. That is, if the *smaller* market has the more elastic demand, it must have the lower price. Thus, the intuition of the separate markets carries over to the more general case under these additional assumptions.

Welfare effects

Much of the discussion about third-degree price discrimination has to do with the welfare effects of allowing this form of price discrimination. Would we generally expect consumer's plus producer's surplus to be higher or lower when third-degree price discrimination is present than when it is not?

We begin with formulating a general test for welfare improvement. Suppose for simplicity that there are only two groups and start with an aggregate utility function of the form $u(x_1, x_2) + y$. Here x_1 and x_2 are the consumptions of the two groups and y is money to be spent on other consumption goods. The inverse demand functions for the two goods are given by

$$p_1(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_1}$$

$$p_2(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_2}.$$

We assume that $u(x_1, x_2)$ is concave and differentiable, though this is slightly stronger than needed.

Let $c(x_1, x_2)$ be the cost of providing x_1 and x_2 , so that social welfare is measured by

$$W(x_1, x_2) = u(x_1, x_2) - c(x_1, x_2).$$

Now consider two configurations of output, (x_1^0, x_2^0) and (x_1', x_2') , with associated prices (p_1^0, p_2^0) and (p_1', p_2') . By the concavity of $u(x_1, x_2)$, we have

$$u(x_1', x_2') \leq u(x_1^0, x_2^0) + \frac{\partial u(x_1^0, x_2^0)}{\partial x_1}(x_1' - x_1^0) + \frac{\partial u(x_1^0, x_2^0)}{\partial x_2}(x_2' - x_2^0).$$

Rearranging and using the definition of the inverse demand functions, we have

$$\Delta u \leq p_1^0 \Delta x_1 + p_2^0 \Delta x_2.$$

By an analogous argument, we have

$$\Delta u \geq p'_1 \Delta x_1 + p'_2 \Delta x_2.$$

Since $\Delta W = \Delta u - \Delta c$, we have our final result:

$$p_1^0 \Delta x_1 + p_2^0 \Delta x_2 - \Delta c \geq \Delta W \geq p'_1 \Delta x_1 + p'_2 \Delta x_2 - \Delta c. \quad (14.19)$$

In the special case of constant marginal cost, $\Delta c = c\Delta x_1 + c\Delta x_2$, so the inequality becomes

$$(p_1^0 - c)\Delta x_1 + (p_2^0 - c)\Delta x_2 \geq \Delta W \geq (p'_1 - c)\Delta x_1 + (p'_2 - c)\Delta x_2. \quad (14.20)$$

Note that these welfare bounds are perfectly general, based only on the concavity of the utility function, which is, in turn, basically the requirement that demand curves slope down. Varian (1985) derived the inequalities using the indirect utility function, which is slightly more general.

In order to apply these inequalities to the question of price discrimination, let the initial set of prices be the constant monopoly prices so that $p_1^0 = p_2^0 = p^0$, and let (p'_1, p'_2) be the discriminatory prices. Then the bounds in (14.20) become

$$(p^0 - c)(\Delta x_1 + \Delta x_2) \geq \Delta W \geq (p'_1 - c)\Delta x_1 + (p'_2 - c)\Delta x_2. \quad (14.21)$$

The upper bound implies that *a necessary condition for welfare to increase is that total output increase*. Suppose to the contrary that total output decreased so that $\Delta x_1 + \Delta x_2 < 0$. Since $p^0 - c > 0$, (14.21) implies that $\Delta W < 0$. The lower bound gives a sufficient condition for welfare to increase under price discrimination, namely that the sum of the *weighted output* changes is positive, with the weights being given by price minus marginal cost.

The simple geometry of the bounds is shown in Figure 14.4. The welfare gain ΔW is the indicated trapezoid. The area of this trapezoid is clearly bounded above and below by the area of the two rectangles.

As a simple application of the welfare bounds, let us consider the case of two markets with linear demand curves,

$$\begin{aligned} x_1 &= a_1 - b_1 p_1 \\ x_2 &= a_2 - b_2 p_2. \end{aligned}$$

For simplicity set marginal costs equal to zero. Then if the monopolist engages in price discrimination, he will maximize revenue by selling halfway down each demand curve, so that $x_1 = a_1/2$ and $x_2 = a_2/2$.

Now suppose that the monopolist sells at a single price to both markets. The total demand curve will be

$$x_1 + x_2 = a_1 + a_2 - (b_1 + b_2)p.$$

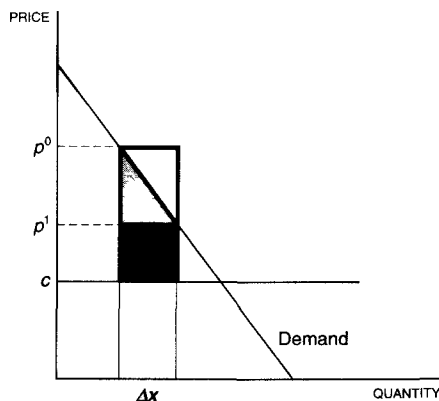


Figure 14.4 **Illustration of the welfare bounds.** The trapezoid is the true change in consumer's surplus.

To maximize revenue the monopolist will operate halfway down the demand curve which means that

$$x_1 + x_2 = \frac{a_1 + a_2}{2}.$$

Hence, with linear demand curves the total output is the same under price discrimination as under ordinary monopoly. The bound given in (14.21) then implies that welfare must decrease under price discrimination.

However, this result relies on the assumption that both markets are served under the ordinary monopoly. Suppose that market 2 is very small, so that the profit-maximizing firm sells zero to this market if price discrimination is not allowed, as illustrated in Figure 14.5.

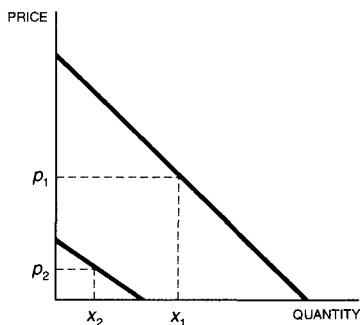


Figure 14.5 **Price discrimination.** Here the monopolist would optimally choose to serve only the large market if it could not price discriminate.

In this case allowing price discrimination results in $\Delta x_1 = 0$ and $\Delta x_2 > 0$, providing an unambiguous welfare gain by (14.21). Of course, this is not only a welfare gain, but is in fact a Pareto improvement.

This example is quite robust. If a new market is opened up because of price discrimination—a market that was not previously being served under the ordinary monopoly—then we will typically have a Pareto improving welfare enhancement. On the other hand, if linearity of demand is not a bad first approximation, and output does not change too drastically in response to price discrimination, we might well expect that the net impact on welfare is negative.

Notes

The discussion of quality choice is based on Spence (1975). For a survey of price discrimination see Varian (1989a).

Exercises

14.1. The inverse demand curve is given by $p(y) = 10 - y$, and a monopolist has a fixed supply of 4 units of a good available. How much will it sell and what price will it set? What would be the price and output in a competitive market with these demand and supply characteristics? What would happen if the monopolist had 6 units of the good available? (Assume free disposal.)

14.2. Suppose that a monopolist faces a demand curve of $D(p) = 10 - p$ and has a fixed supply of 7 units of output to sell. What is its profit-maximizing price and what are its maximal profits?

14.3. A monopolist faces a demand curve of the form $x = 10/p$, and has a constant marginal cost of 1. What is the profit maximizing level of output?

14.4. For what form of demand curve does $dp/dc = 1$?

14.5. Suppose that the inverse demand curve facing a monopolist is given by $p(y, t)$, where t is a parameter that shifts the demand curve. For simplicity, assume that the monopolist has a technology that exhibits constant marginal costs. Derive an expression showing how output responds to a change in t . How does this expression simplify if the shift parameter takes the special form $p(y, t) = a(y) + b(t)$?

14.6. The demand function facing the monopolist is given by $D(p) = 10/p$, and the monopolist has positive marginal cost of c . What is the profit-maximizing level of output?

14.7. Suppose marginal costs are constant at $c > 0$ and that the demand function is given by

$$D(p) = \begin{cases} 10/p & \text{if } p \leq 20 \\ 0 & \text{if } p > 20. \end{cases}$$

What is the profit-maximizing price?

14.8. For what form of utility function and demand curve does the monopolist produce the optimal level of quality, given its quantity choice?

14.9. In the text we gave a graphical argument that if $\partial^2 p / \partial x \partial q > 0$ then $\partial u / \partial q - x \partial p / \partial q < 0$. Let us prove this algebraically. Here are the steps to follow: 1) Show that the hypothesis implies that if $z < x$ then

$$\frac{\partial p(z, q)}{\partial q} < \frac{\partial p(x, q)}{\partial q}.$$

2) Express the left-hand side of the inequality in terms of the utility function. 3) Integrate both sides of this inequality over z ranging from 0 to x .

14.10. One common way to price discriminate is to charge a lump sum fee to have the *right* to purchase a good, and then charge a per-unit cost for consumption of the good after that. The standard example is an amusement park where the firm charges an entry fee and a charge for the rides inside the park. Such a pricing policy is known as a **two part tariff**. Suppose that all consumers have identical utility functions given by $u(x)$ and that the cost of providing the service is $c(x)$. If the monopolist sets a two part tariff, will it produce more or less than the efficient level of output?

14.11. Consider the graphical treatment of the second-degree price discrimination problem. Look at Figure 14.3C carefully and answer the following question: under what conditions would the monopolist sell only to the high-demand consumer?

14.12. If the monopolist chooses to sell to both consumers, show that area B must be less than area A .

14.13. Suppose that there are two consumers who each may purchase one unit of a good. If the good is of quality q , consumer t achieves utility $u(q, t)$. It costs the monopolist zero to provide quality. Let the maximum price that consumer t would be willing to pay for quality q be given by w_t . The monopolist cannot distinguish the two consumers and must therefore offer at most two different qualities between which the consumers can freely choose. Set up the profit maximization problem for the monopolist, and analyze it thoroughly. Hint: does this problem look like anything you've seen before?

14.14. The monopolist can also be thought of as choosing the price and letting the market determine how much is sold. Write down the profit maximization problem and verify that $p[1 + 1/\epsilon] = c'(y)$ at the optimal price.

14.15. There is a single monopolist whose technology exhibits constant marginal costs, i.e., $c(y) = cy$. The market demand curve exhibits constant elasticity, ϵ . There is an *ad valorem* tax on the price of the good sold so that when the consumer pays a price P_D , the monopolist receives a price of $P_S = (1 - \tau)P_D$. (Here P_D is the demand price facing the consumer and P_S is the supply price facing the producer.)

The taxing authority is considering changing the *ad valorem* tax to a tax on output, t , so that we will have $P_D = P_S + t$. You have been hired to calculate the output tax t that is equivalent to the *ad valorem* tax τ in the sense that the final price facing the consumer is the *same* under either scheme.

14.16. Suppose that the inverse demand curve facing a monopolist is given by $p(y, t)$, where t is a parameter that shifts the demand curve. For simplicity, assume that the monopolist has a technology that exhibits constant marginal costs.

(a) Derive an expression showing how output responds to a change in t .

(b) How does this expression simplify if the inverse demand function takes the special form $p(y, t) = a(y) + b(t)$?

14.17. Consider a simple economy which acts as though there is one consumer with utility function $u_1(x_1) + u_2(x_2) + y$, where x_1 and x_2 are the amounts of goods 1 and 2, respectively, and y is money to spend on all other goods. Suppose that good 1 is supplied by a firm that acts competitively and good 2 is supplied by a firm that acts like a monopoly. The cost function for good i is denoted by $c_i(x_i)$, and there is a specific tax of amount t_i on the output of industry i . Assume that $c_i' > 0$, $p_i' < 0$, and $p_i'' < 0$.

(a) Derive expressions for dx_i/dt_i for $i = 1, 2$ and sign them.

(b) Given a change in outputs (dx_1, dx_2) , derive an expression for the change in welfare.

(c) Suppose that we consider taxing one of the two industries and using the proceeds to subsidize the other. Should we tax the competitive industry or the monopoly?

14.18. There are two consumers who have utility functions

$$\begin{aligned}u_1(x_1, y_1) &= a_1x_1 + y_1 \\ u_2(x_2, y_2) &= a_2x_2 + y_2.\end{aligned}$$

The price of the y-good is 1, and each consumer has a “large” initial wealth. We are given that $a_2 > a_1$. Both goods can only be consumed in nonnegative amounts.

A monopolist supplies the x-good. It has zero marginal costs, but has a capacity constraint: it can supply at most 10 units of the x-good. The monopolist will offer at most two price-quantity packages, (r_1, x_1) and (r_2, x_2) . Here r_i is the cost of purchasing x_i units of the good.

(a) Write down the monopolist’s profit maximization problem. You should have 4 constraints plus the capacity constraint $x_1 + x_2 \leq 10$.

(b) Which constraints will be binding in optimal solution?

(c) Substitute these constraints into the objective function. What is the resulting expression?

(d) What are the optimal values of (r_1, x_1) and (r_2, x_2) ?

14.19. A monopolist sells in two markets. The demand curve for the monopolist’s product is $x_1 = a_1 - b_1p_1$ in market 1 and $x_2 = a_2 - b_2p_2$ in market 2, where x_1 and x_2 are the quantities sold in each market, and p_1 and p_2 are the prices charged in each market. The monopolist has zero marginal costs. Note that although the monopolist can charge different prices in the two markets, it must sell all units within a market at the same price.

(a) Under what conditions on the parameters (a_1, b_1, a_2, b_2) will the monopolist optimally choose not to price discriminate? (Assume interior solutions.)

(b) Now suppose that the demand functions take the form $x_i = A_i p_i^{-b_i}$, for $i = 1, 2$, and the monopolist has some constant marginal cost of $c > 0$. Under what conditions will the monopolist choose not to price discriminate? (Assume interior solutions.)

14.20. A monopolist maximizes $p(x)x - c(x)$. In order to capture some of the monopoly profits, the government imposes a tax on revenue of an amount t so that the monopolist’s objective function becomes $p(x)x - c(x) - tp(x)x$. Initially, the government keeps the revenue from this tax.

(a) Does this tax increase or decrease the monopolist’s output?

(b) Now government decides to award the revenue from this tax to the consumers of the monopolist's product. Each consumer will receive a "rebate" in the amount of the tax collected from his expenditures. The representative consumer who spends px receives a rebate of tpx from the government. Assuming quasilinear utility, derive an expression for the consumer's inverse demand as a function of x and t .

(c) How does the monopolist's output respond to the tax-rebate program?

14.21. Consider a market with the following characteristics. There is a single monopolist whose technology exhibits constant marginal costs; i.e.,

$$c(y) = cy.$$

The market demand curve exhibits constant elasticity, ϵ . There is an *ad valorem* tax on the price of the good sold so that when the consumer pays a price P_D , the monopolist receives a price of $P_S = (1 - \tau)P_D$. (Here P_D is the demand price facing the consumer and P_S is the supply price facing the producer.)

The taxing authority is considering changing the *ad valorem* tax to a tax on output, t , so that we will have $P_D = P_S + t$. You have been hired to calculate the output tax t that is equivalent to the *ad valorem* tax τ in the sense that the final price facing the consumer is the *same* under either scheme.

14.22. A monopolist has a cost function of $c(y) = y$ so that its marginal costs are constant at \$1 per unit. It faces the following demand curve:

$$D(p) = \begin{cases} 0, & \text{if } p > 20; \\ 100/p & \text{if } p \leq 20. \end{cases}$$

(a) What is the profit-maximizing choice of output?

(b) If the government could set a price ceiling on this monopolist in order to force it to act as a competitor, what price should they set?

(c) What output would the monopolist produce if forced to behave as a competitor?

14.23. An economy has two kinds of consumers and two goods. Type A consumers have utility functions $U(x_1, x_2) = 4x_1 - (x_1^2/2) + x_2$ and Type B consumers have utility functions $U(x_1, x_2) = 2x_1 - (x_1^2/2) + x_2$. Consumers can only consume nonnegative quantities. The price of good 2 is 1 and all consumers have incomes of 100. There are N type A consumers and N type B consumers.

(a) Suppose that a monopolist can produce good 1 at a constant unit cost of c per unit and cannot engage in any kind of price discrimination. Find its optimal choice of price and quantity. For what values of c will it be true that it chooses to sell to both types of consumers?

(b) Suppose that the monopolist uses a “two-part tariff” where a consumer must pay a lump sum k in order to be able to buy anything at all. A person who has paid the lump sum k can buy as much as he likes at a price of p per unit purchased. Consumers are not able to resell good 1. For $p < 4$, what is the highest amount k that a type A is willing to pay for the privilege of buying at price p ? If a type A does pay the lump sum k to buy at price p , how many units will he demand? Describe the function that determines demand for good 1 by type A consumers as a function of p and k . What is the demand function for good 1 by type B consumers? Now describe the function that determines total demand for good 1 by all consumers as a function of p and k .

(c) If the economy consisted only of N type A consumers and no type B consumers, what would be the profit-maximizing choices of p and k ?

(d) If $c < 1$, find the values of p and k that maximize the monopolist's profits subject to the constraint that both types of consumers buy from it.