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Vote splitting, reelection and electoral control: Towards a unified model

Mauricio Soares Bugarin

Department of Economics, University of Brasilia, Asa Norte, 70910-900, Brasilia, DF, Brazil (e-mail: bugarin@unb.br)

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Dedicated to Hélio Suppo Ribeiro, *in memoriam*

Abstract. This article presents a dynamic game theoretic model of voting in the presence of asymmetric information about a relevant parameter of the economy, the state of the world. Voters may use both vote splitting and reelection as mechanisms of electoral control. In a perfect Bayesian equilibrium, voters will reelect an Executive incumbent if a minimum level of social outcome, n^*, is attained. The main findings are that voters tend to be more demanding, requiring a higher value for n^*, if they expect the true state of the world to be *favorable*, and less demanding if they believe the state of the world is *unfavorable*. Moreover, vote splitting will be chosen if a favorable state is expected, whereas if an unfavorable state is more likely, voters reduce pressure over the incumbent by choosing a unified government.

"... c'est une expérience éternelle, que tout homme qui a du pouvoir est porté à en abuser; il va jusqu'à ce qu'il trouve des limites. Qui le dirait! la vertu même a besoin de limites."
Montesquieu, *De l'esprit des lois*, Book XI, Chapt. IV

1 Introduction

Delegation of power is an unavoidable consequence of organization in societies. Indeed, as population increases, so does the number and complexity of

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the decisions to be made and it becomes impossible for a society to make those decision in a board composed by all of its members.

However, many problems emerge when a few representatives have to decide for an entire nation. Despotism, corruption, inefficiency and exclusion are among the most important ones. Since Montesquieu (1748)'s analysis of the adverse consequences of concentration of power, modern societies have developed sophisticated rules to deal with the problems of delegation. The separation of powers with a system of checks and balances, universal suffrage and term limits, for example, are all mechanisms created in order to induce those who are given the power from society to act in its best interest.

With the advent of the economics of uncertainty and information a new field of research has emerged. That research focuses on the instruments available to society to control the behavior of its representatives in a democracy, and how society uses those instruments.

The traditional approach emphasizes **reelection** as the main source of societal control. The two seminal articles using this approach are Barro (1973) and Ferejohn (1986). Barro uses a *Principal-Agent* approach in a multi-period finite game in which voters are the Principals and an executive officeholder is the Agent. Voters elect the officeholder in the first period and the incumbent chooses the level of government spending on a public good. The incumbent executive extracts "political income" from payments above the competitive factors of production prices. Therefore, the incumbent has an incentive to spend above the level that would be optimal to society. Barro shows that reelection can be used as an effective instrument to reduce the incumbent's overpayments and that the higher the number of successive terms allowed, the better to the voters.

Barro's article shows the gains of electoral control that can be obtained from reelection in the context of a finite-horizon complete-information model. Ferejohn studies a model where there is asymmetric information in the context of an infinitely repeated game. Once again voters are the Principals who elect an executive officeholder, the Agent. The state of the world, which can be interpreted as the officeholder's (macroeconomic) performance, depends on an effort choice that is taken by the officeholder and is costly. Moreover, the state of the world is also influenced by a stochastic component which is observed only by that incumbent. Therefore, while voters would like the incumbent to choose a high effort level, they only observe the combined result of the incumbent effort *and* the stochastic component of the state of nature. At the end of each period of this infinite game, voters observe the state of nature and decide whether or not to reelect the incumbent. As in Barro's model, voters optimal reelection strategy will induce the incumbent into a higher level of effort than he would choose if reelection was not allowed.

Although extremely insightful and path breaking, the above articles totally exclude an important interaction that is present in the real-world political arena: the executive is not the only responsible for the political decisions that are taken. In fact, every political decision is typically the result of a delicate negotiation process between the executive and the legislative branches of gov-

ernment. A more recent class of models focuses on the relationship between the executive and the legislative powers and points out **vote splitting** as a fundamental variable that enters voters analysis when taking their electoral decisions. The seminal references in this second approach are Fiorina (1988)[1] and Alesina and Rosenthal (1995)[2].

Fiorina's article assumes that there are two parties which have fixed ideological positions. The ideologies of the parties tend to be extreme opposites in a one-dimensional policy space. Moreover, voters have their own ideological positions on that same policy space. Finally, the interaction between Congress and the President is modeled as follows. If one party controls both institutions, that party's policy is implemented; however, when one party controls the Legislature and the other party controls the executive, a more moderate policy ensues, as an (exogenously defined) consequence of bargaining between the two powers. Therefore, voters whose ideological positions are in the "middle" of the policy space will choose to split their political ticket, resulting in a divided government. Hence, vote splitting appears as an important mechanism voters may use to increase electoral control, in the presence of ideological parties.

Alesina and Rosenthal present a more carefully formalized model in which the policy that results from the negotiation between the executive and the legislative depends on which party control each one of those powers – as in Fiorina's model – but it also depends on the *strength* of each party in Congress. Indeed, the higher the proportion of the representatives of a party in Congress, the more it influences outcome towards its preferred policy. Moreover, voters are assumed to have a more sophisticated voting behavior, called "conditionally sincere voting", in which voters choose their tickets based on the expected decisions of the other voters. This more refined approach to the strategic choice of voters is shown to increase the likelihood of vote splitting.

Finally, Bugarin (1999) adds to the above structures a precise bargaining model that endogenizes the policy outcome of the bargaining process, and finds that vote splitting may be a natural choice that voters make when they are uncertain about the underlying state of nature, in an institutional framework characterized by opposite ideological parties.

As it can be noted by this short review of the main models of electoral behavior, both **reelection** *and* **vote splitting** can be important control instruments of elected representatives. Moreover, in real-world elections voters are faced with those two instruments when they have to take their ballots. However, given the sophistication of the theoretic modeling involved, the literature has tended to study those mechanisms in the two separate classes of models mentioned above, failing to analyze them in an integrated framework.

The present article aims at filling that gap between economic modeling and empirical evidence, by presenting a game theoretic model of voters' behavior in which voters elect the Executive and the Legislature, choosing

[1] See also Fiorina (1992) and (1996).
[2] See also Alesina and Rosenthal (1989) and (1996).

from candidates of different parties. Once elected, the Executive passes a budget from which he/she can extract "political income" in the form of overspending. The real economic cost associated to the budget is revealed only to the Executive and can take one of many values, depending on the state of the world. Passing an expensive budget in the Legislature is costly to the Executive; moreover, this cost is higher if the Executive incumbent's party does not hold the majority of seats. Voters decide the composition of the Legislature and the Executive in the first period, observe the outcome of the political bargaining process between the two powers, then decide whether to reelect the Executive and/or the Legislative representatives.

The model shows that voters' behavior is totally determined by their beliefs about the state of the world. If voters believe that the economy is in a good state, then only an Executive presenting a budget with a small amount of overspending will be reelected. Moreover, voters will split their ticket by choosing an Executive officeholder from a party and a majority in the Legislature from a different party. On the other hand, if voters believe that the economy is likely to be in a bad state, then an Executive incumbent passing a relatively more expensive budget will be reelected and his/her party will be given majority in the Legislature.

The findings of the model confirm a puzzling observation. Indeed, on one hand, a country under severe crisis sometimes finds a hardly understandable political unity, whereas, on the other hand, voters may be strongly divided in their choices of Executive and Legislative representatives in a country experiencing a clear expansion path.

The rest of the paper is divided as follows. Next section presents the basic general model of voting behavior in the presence of uncertainty, asymmetric information and the mechanisms of reelection and vote splitting. Section 3 presents a simplification in the dimensionality of the policy space that allows a tractable solution. Section 4 solves the model. Section 5 presents two possible scenarios, depending on the ex-ante expectation of voters regarding the underlying state of nature. Section 6 analyses the 1998 presidential elections in Brazil in the light of the model studied in the present article. Section 7 presents a discussion on the bargaining cost function between Executive and the Legislature. Finally, Sect. 8 presents some concluding remarks.

2 The model

There are two periods. Voters elect the Executive and the Legislature at each period. Once elected, the Executive incumbent proposes a budget of the form $(N, P) = \{(n_1, p_1), \ldots, (n_k, p_k)\}$ where p_i is the proposed unitary cost of project i and n_i is the proposed number of copies of the project to be implemented, $i = 1, \ldots, k$. Then the Executive officeholder and the legislators bargain over the proposed project.

Each project i has a real cost r_i, corresponding to competitive factors of production, which is the private information of the proposer. If $p_i > r_i$, then

there is an overpayment $e_i = p_i - r_i$ from which the Executive extracts political income. A factor $\beta_i \in (0,1)$ describes this political income in that $n_i\beta_i e_i$ is the utility gain to the Executive from project (n_i, p_i). The coefficient β_i can be though to be the Executive corruption factor associated to project (n_i, p_i). The choice of proposal (N, P) is restricted by a budget constraint which requires total spending not to be higher than total available resources[3] B, i.e., $\sum_{i=1}^{k} n_i p_i \leq B$.

Voters elect the Executive incumbent and the Legislators from two identical parties, I and II. The bargaining process among elected officials is summarized by a cost function c to the proposer, which depends both on the proposed costs p_i, $i = 1, \ldots, k$ and the representation $1 - \pi$ of the Executive incumbent's party in the Legislature, i.e., $c = c(p_1, \ldots, p_k, \pi)$ where π is the proportion of the opposition to the Executive incumbent's party in the Legislature.

Therefore, if the incumbent passes a budget $(N, P) = \{(n_1, p_1), \ldots, (n_k, p_k)\}$, her resulting utility is:

$$v(N, P, \pi) = \sum_{i=1}^{k} n_i\beta_i(p_i - r_i) - c(p_1, \ldots, p_k, \pi).$$

The cost function c is assumed to be differentiable and satisfy the following properties:

(c-1) $c(0, \ldots, 0, \pi) = 0$, $\forall \pi \in [0,1]$.

(c-2) For every $\pi \in [0,1]$, $c(p_{-i}, \cdot, \pi)$ is a strictly increasing, strictly convex function of p_i, $i = 1, \ldots, k$.

(c-3) For every $p \in \mathbb{R}_+^k$, $c(p, \cdot)$ is a strictly increasing function of π.

(c-4) There exists a point $p^* \in \mathbb{R}_+^k$ such that $\dfrac{\partial}{\partial \pi} c(\cdot, \pi)$ is an increasing function of p for $p < p^*$ and a decreasing function of p for $p > p^*$.

(c-5) For every $p \in \mathbb{R}_+^k$, $\dfrac{\partial}{\partial p_i} c(p, \cdot)$ is a strictly increasing function of π.

Condition (c-1) states that there is no cost in passing no project, regardless of the opposition in the Legislature. Condition (c-2) says that the higher the unitary cost of a project the higher the opposition to it in the Legislature and thus, the higher its approval cost. Moreover, the rate in which the opposition to a costly project increases is also increasing in its cost.

Condition (c-3) models the fact that the higher the representation of the opposing party in the Legislature, the costlier for the Executive to pass a budget.

Condition (c-4) intends to model the evolution of beliefs, about the true cost of a project, in the Legislature. Although opposition increases both when

[3] Total available resources may include some level of fiscal deficit, so that a budget constraint does not necessarily requires a balanced budget.

proposed costs and opposition representation increase, the rate of increase of c as the opposition representation grows, decreases after a threshold value p^*. Notice that the Legislators still oppose more costly projects, but the *impetus* of this opposition decreases after a certain point, since Legislators start believing that only a really costlier project would generate such costly proposals.

Finally, condition (c-5) states that the rate at which the opposition of a project increases as its cost increases, is higher when the opposition party's representation in the Legislature augments.

Voters are uncertain about the true cost of the projects, $r = (r_1, \ldots, r_k)$, but know that they must be one of two possible values: $r = l$ or $r = h$ with $l < p^* < h$. The cost $r = l$ corresponds to a "good" state of the world, in that it is cheap to implement the projects, whereas $r = h$ corresponds to a "bad" state in that the projects are costly. Voters assign probability $\rho \in (0, 1)$ for the state $r = l$ and $1 - \rho$ for $r = h$.

The uncertainty about the state of the world complicates voters' decisions at period 2: suppose the Executive incumbent has passed an expensive project at period 1. Does this mean the incumbent extracted too much political income in a good state of the world, in which case he should not be reelected, or does this mean that true costs are really high and the incumbent chose a minimal overpayment, in which case, he should be reelected? Symmetrically, this uncertainty may allow the Executive incumbent to get extra political income in the good state, by pretending it is in fact a bad state.

At each period, voters derive utility from the social return of the implemented budget (N, P), which is measured in terms of the number of projects implemented. This social return is given by the real-valued function $\varphi(N) = \varphi(n_1, \ldots, n_k)$, which is assumed to be concave and strictly increasing with $\varphi(0) = 0$. Therefore, voters want to induce the Executive incumbent to pass a budget with high values of N; they will devise a mechanism that will use optimally both reelection and vote splitting in order to achieve that goal, as will become clear in the following sections.

3 The one-dimensional approach

In order to simplify the analysis of the game, assume that $k = 1$, i.e., there is one project to be implemented and politicians have to agree on how many units of the project, n, to implement and how much money p to allocate to each unit, given the budget constraint B.[4]

Then the budget constraint becomes $np = n(r + e) \leq B$, where the index $i = 1$ has been dropped. Furthermore, the period-specific utility of the Execu-

[4] This technical simplification may be appropriate in a local election polarized on one issue; suppose, for example, that a local community has allocated a certain maximum amount to build housing for the homeless and that the relevant issue for the election is how many units to build.

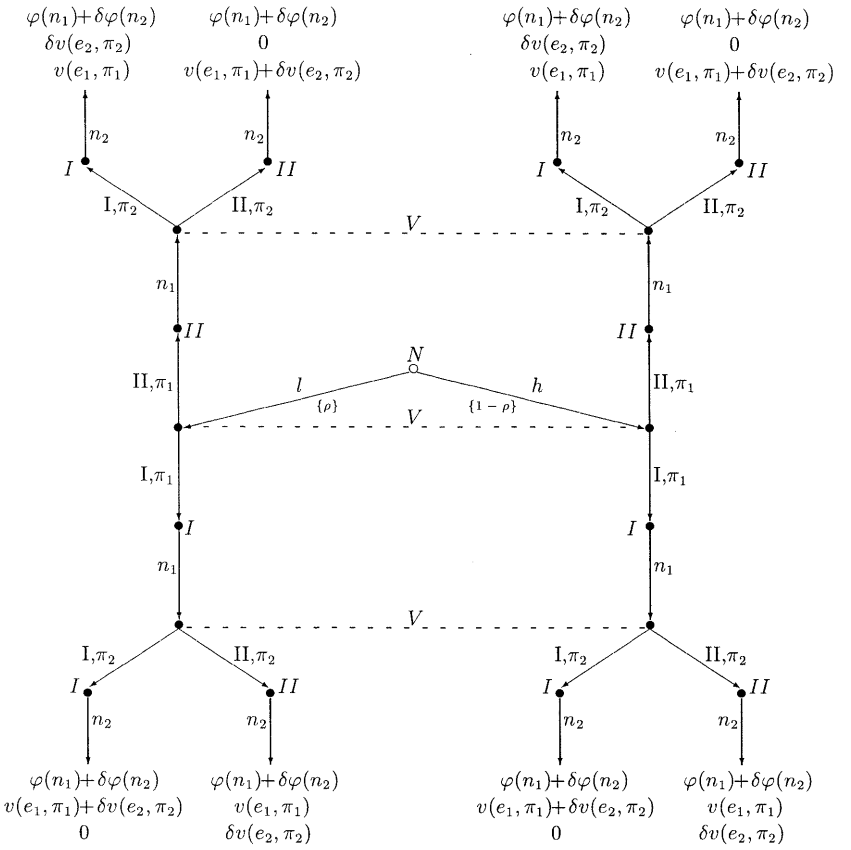

Fig. 1. The extensive form game

tive becomes:

$$v(n, p, \pi) = n\beta(p - r) - c(p, \pi)$$

Note that a proposal (n, p) with $np < B$ is strictly dominated by the feasible proposal (n', p) where $n' = B/p$, i.e., in equilibrium the budget constraint holds with equality. Therefore, writing $e = p - r$, the Executive incumbent's utility can simply be written as:

$$v(r, e, \pi) = \beta B \frac{e}{r + e} - c(r + e, \pi)$$

Figure 1 presents an extensive form of the game played by the voters and the politicians. At the beginning of period 1 the state of the world r is realized but not observed by the voters, who elect the Executive incumbent (from party I or II) and the proportion of the opposing party in the Legislature (π_1). The elected Executive observes r, chooses the overspending value e_1 and passes the project (n_1, p_1) where $p_1 = p(e_1) = r + e_1$ and $n_1 = n(e_1) = \dfrac{B}{r + e_1}$. Since

$p_1 = B/n_1$, it is eliminated from the game tree. The index 1 refers now to the first period.

Period 2 starts with voters deciding whether to reelect the Executive incumbent or not, as well as the proportion of the opposition party in the Legislature, π_2. Then the elected Executive chooses e_2 and passes the project (n_2, p_2) where $p_2 = r + e_2$ and $n_2 = \dfrac{B}{r + e_2}$. Finally, the game concludes.

The utility of a representative voter is $\varphi(n_1) + \delta\varphi(n_2)$ where $\delta \in (0, 1)$ is all agents' intertemporal discount factor.

The utility of a reelected Executive incumbent is $v(e_1, \pi_1) + \delta v(e_2, \pi_2)$. The utility of an incumbent that is elected only at period 1 is $v(e_1, \pi_1)$ and the present value of the utility of an Executive that is elected only at period 2 is $\delta v(e_2, \pi_2)$.

For simplicity, the game tree presents one generic choice among infinitely many at each node; for example, when voters (V) elect an incumbent of party I at period 1, they can choose any proportion of party II representatives in the Legislature: $\pi_1 \in [0, 1]$.

4 Solving the game

4.1 The second period

The natural solution concept for this model is perfect Bayesian equilibrium, given the existence of imperfect information. Let us start solving the game by sequential rationality. At period 2, an elected incumbent solves the following maximization problem.

$$\max_e v(r, e, \pi_2) = \beta B \frac{e}{r + e} - c(r + e, \pi_2).$$

Notice that $f(e) = \beta B \dfrac{e}{r + e}$ is strictly concave and $g(e) = c(r + e, \pi_2)$ is strictly convex; moreover, $f'((0, +\infty)) = (0, \beta B/r]$. Therefore, if for each $\pi_2 \in [0, 1]$ there exists $e > 0$ such that $g'(e) > B/r$ then is a unique solution, $\hat{e}(r, \pi_2)$ to the above problem[5]. The incumbent's choice is then:

$$\hat{n}_2(r, \pi_2) = \frac{B}{r + \hat{e}(r, \pi_2)} \qquad \text{if } v(r, \hat{e}(r, \pi_2), \pi_2) \geq 0,$$

$$\hat{n}_2(r, \pi_2) = 0 \text{ (no proposal at all)} \quad \text{otherwise.}$$

[5] Since the objective function is strictly concave, there will be either a unique solution or no solution at all to the problem. A simpler condition that ensures the existence of a solution is to assume that $\dfrac{\partial c}{\partial p}(\cdot, \pi_2)$ is unbounded for every π_2. The author is indebted to Wilfredo Maldonado for pointing out this sufficient condition, which is assumed to hold hereforth.

This study assumes that $v(r, \hat{e}(r, \pi_2), \pi_2) > 0$, $r = l, h$, i.e., the political income that an officeholder receives is always big enough to strictly compensate the cost of passing an optimal project in the Legislature.

The following proposition characterizes voters' choice of π_2. The proofs to all propositions of this article are relegated to the Appendix.

Proposition 1. *The optimal corruption level $\hat{e}(r, \pi_2)$ of an incumbent at period 2 is a decreasing function of π_2. In particular, voters will chose $\pi_2 = 1$ at period 2.*

The above result follows immediately from the first order conditions of the incumbent maximization problem at period 2. An important consequence of this result is that it is optimal to the voters to reinforce the opposing party in the Legislature at the last period, i.e., vote splitting at $t = 2$ is optimal regardless of the state of nature. That result is intuitive: at the last period reelection cannot be used in order to induce a more favorable outcome, therefore vote splitting becomes the only control mechanism voters are left with.

Notice that voters are indifferent at period 2 between an Executive from party I and legislators from party II, or an Executive from party II with legislators from party I. Therefore, they can make their decisions of reelecting an incumbent contingent on that incumbent's choice n_1 at period 1. Given this strategic opportunity, voters' optimal reelection strategy will be given by a threshold number n^* such that the Executive incumbent is reelected if and only if $n_1 \geq n^*$.

4.2 The first period

Given that parties are essentially identical, any choice for the incumbent's party at period 1 is optimal, as long as π_1 and n^* are chosen properly. The choice of π_1 and n^* will depend upon voters' beliefs ρ about the state of the world, as described below.

Suppose voters could observe r. Then voters would be able to induce a minimal level of overpayment $\tilde{e}(r, \pi_1)$ by setting $n^* = \dfrac{B}{r + \tilde{e}(r, \pi_1)}$ where $\tilde{e}(r, \pi)$ is the minimal value of e such that:

$$v(r, e, \pi_1) + \delta v(r, \hat{e}(r, 1), 1) = v(r, \hat{e}(r, \pi_1), \pi_1)$$

That is, in the above equation e is chosen in such a way that an incumbent is indifferent between choosing her optimal $\hat{e}(r, \pi_1)$ at period $t = 1$ and not being reelected or choosing the lower[6] $\tilde{e}(r, \pi_1)$ at $t = 1$, being reelected, and choosing $\hat{e}(r, 1)$ at the second period. By differentiating the above equation and using the properties of the cost function c, the following proposition can be proved.

Proposition 2. *If voters could observe the true state of the world and if $\hat{e}(l, 0)$ is small enough compared to the value of l,[7] then:*

[6] By construction $v(r, e, \pi)$ is a strictly concave, nonmonotonic function of e, hence it is single peaked.

[7] More precisely, it must be the case that $l + \hat{e}(l, 0) < p^*$.

(i) *When the economy is in a bad state, i.e., $r = h$, the function $\tilde{e}(r, \pi_1)$ is minimized by choosing $\pi_1 = 0$. Therefore, in a bad state of the world, voters would prefer a unified government.*

(ii) *When the economy is in a good state, i.e., $r = l$, the function $\tilde{e}(r, \pi_1)$ is minimized by choosing $\pi_1 = 1$. Therefore, in a good state of the world, voters would prefer a divided government.*

Since voters do not observe r, their decisions will depend on the likelihood of the different states of the world, described by ρ, as well as the parameters:

$$\tilde{n}_l = \frac{B}{l + \tilde{e}(l,1)}, \quad \hat{n}_l = \frac{B}{l + \hat{e}(l,1)}, \quad \tilde{n}_h = \frac{B}{h + \tilde{e}(h,0)}, \quad \hat{n}_h = \frac{B}{h + \hat{e}(h,0)}$$

A simple comparison using the information available shows the following result.

Proposition 3. *The above threshold values for the number of projects (n) satisfy the following ordering:*

$$\hat{n}_h < \tilde{n}_h < \hat{n}_l < \tilde{n}_l$$

The graph below shows voters utilities according for each possible value of n.

Note that voters will have some control over the Executive incumbent regardless of the state of the world if $n^* \in [\hat{n}_h, \tilde{n}_h]$. In that case an incumbent will choose n^* in the bad state of the world and $\hat{n}_l$ in the good state. Both incumbents will be reelected and voters' expected utility in the first period will be $\rho\varphi(\hat{n}_l) + (1-\rho)\varphi(n^*)$. Therefore, any choice of $n^* \in [\hat{n}_h, \tilde{n}_h)$ is strictly dominated by the choice $n^* = \tilde{n}_h$.

On the other hand, voters will control the Executive only in a good state of the world if $n^* \in [\hat{n}_l, \tilde{n}_l]$. In that case an incumbent will choose $\hat{n}_h$ in a bad state of the world and will not be reelected, and will choose $\tilde{n}_l$ in a good state and will be reelected. Thus, voters' expected utilities in the first period will be $\rho\varphi(n^*) + (1-\rho)\varphi(\hat{n}_h)$. Therefore, any choice of $n^* \in [\hat{n}_l, \tilde{n}_l)$ is strictly dominated by $n^* = \tilde{n}_l$. The following result establishes the optimal choice of n^* as a function of the *ex ante* beliefs ρ.

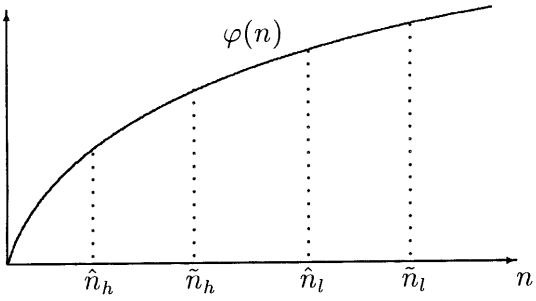

Fig. 2. Voters' utility

Proposition 4. *Suppose that* $l + \hat{e}(l,0) < p^*$. *Then there are exactly two possible perfect Bayesian equilibria to this game, which are described below.*

(i) *If* $\rho > \dfrac{\varphi(\tilde{n}_h) - \varphi(\hat{n}_h)}{(\varphi(\tilde{n}_l) - \varphi(\tilde{n}_h)) - (\varphi(\hat{n}_l) - \varphi(\hat{n}_h))}$, *then in the unique perfect Bayesian equilibrium of the game voters choose divided government in both periods* $(\pi_1 = \pi_2 = 1)$ *and* $n^* = \tilde{n}_l$. *An incumbent will choose* $\hat{n}_h$ *in a bad state of nature and will not be reelected and will choose* $\tilde{n}_l$ *in a good state and will be reelected.*

(ii) *If* $\rho < \dfrac{\varphi(\tilde{n}_h) - \varphi(\hat{n}_h)}{(\varphi(\tilde{n}_l) - \varphi(\tilde{n}_h)) - (\varphi(\hat{n}_l) - \varphi(\hat{n}_h))}$, *then in the unique perfect Bayesian equilibrium of the game voters will choose unified government in the first period* $(\pi_1 = 0)$, *divided government in the second period* $(\pi_2 = 1)$ *and* $n^* = \tilde{n}_h$. *An incumbent will choose* $\tilde{n}_h$ *in a bad state of the world and* $\hat{n}_l$ *in a good state of the world. The incumbent will always be reelected.*

Notice that the equilibria are all of the separating type. In particular, it will be trivial to check for Bayesian consistence of beliefs in the second period, on the equilibrium path. The following section presents an interpretation of the equilibria found in Proposition 4.

5 Two possible scenarios

5.1 The optimistic expectation

Suppose voters believe that they are in a good state of the world with a high probability, i.e.,

$$\rho > \frac{\varphi(\tilde{n}_h) - \varphi(\hat{n}_h)}{(\varphi(\tilde{n}_l) - \varphi(\tilde{n}_h)) - (\varphi(\hat{n}_l) - \varphi(\hat{n}_h))}$$

Then voters will choose $n^* = \tilde{n}_l$. In this case incumbents will choose $n = \tilde{n}_l$ and will be reelected if $r = l$ and will choose $n = \hat{n}_h$ and will not be reelected if $r = h$. Therefore, this is a fully revealing separating equilibrium, in that voters learn the true state of the world after observing the choice of the incumbent at period 1. Moreover, reelection occurs only in the good state of the world. Since voters believe the good state is likely, they require an excellent incumbent's performance in order to reelect him.

Furthermore, in this first scenario the incumbent will always face strong opposition in the Legislature since vote splitting is optimal to the voters in both periods.

5.2 The pessimistic expectation

Suppose now voters believe that they are in a bad state of the world with a high probability, i.e.,

$$\rho < \frac{\varphi(\tilde{n}_h) - \varphi(\hat{n}_h)}{(\varphi(\tilde{n}_l) - \varphi(\tilde{n}_h)) - (\varphi(\hat{n}_l) - \varphi(\hat{n}_h))}$$

Then voters will choose $n^* = \tilde{n}_h$. In this second situation, an incumbent will choose $n^* = \hat{n}_l$ if $r = l$ and $n^* = \tilde{n}_h$ if $r = h$. In both cases the incumbent is reelected. Although this is again a fully revealing equilibrium, in this case the incumbent is always reelected. Since voters believe that a bad state of the world is likely, it is optimal to set a more flexible reelection requirement, which will control optimally the incumbent choice when the state is truly bad and will tolerate a suboptimal output when the revealed state is a good one.

Furthermore, in the second scenario an incumbent will negotiate with a favorable Legislature in the first period, since voters will choose a unified government.

6 An application to the Brazilian 1998 presidential elections

On October 4, 1998 Brazilians left their homes to choose a new president in the midst of huge internal turmoil caused by a speculative attack against its currency, the *real*, by investors who were expatriating their financial assets at an astonishing rate. There were two strong candidates: President Fernando Henrique Cardoso (FHC), who was running for reelection, and Luís Inácio Lula da Silva, the Labor Party candidate. The results were sharp: FHC received 53.06% of valid votes against 31.71% for Lula. Although Brazilian electoral regime requires a second turn with the first two major candidates if none receives at least 50% of all valid votes, FHC's victory was so strong that no second turn was needed.

The electoral campaign, however, was not as clear cut as the final result, as can be seen in the following table, which shows opinion poll results by the *Vox Populi Institute*. Although FHC had the lead since the first polls, Lula's position increased steadily from December 1997 to June 1998, when the gap between the two candidates narrowed from 15% to only 1%.[8] One of the

Table 1. Opinion polls and electoral outcome of the 1998 Brazilian presidential elections

	December 9, '97	May 16, '98	June 4, '98	August 30, '98	Final October
Lula	18%	25%	30%	31%	31.71%
FHC	33%	34%	31%	55%	53.06%

Source: The Vox Populi Institute

[8] According to another opinion research institute, the IBOPE, this difference narrowed from 17% to 5% in the same period.

strongest campaign flags Lula held against FHC was that unemployment was at historically high levels in Brazil since FHC implemented the *Real Plan* to reduce inflation. However, between July and August 1998 the gap widened again, due to a fast increase in FHC's position, culminating in August 30 to a 24% difference.

What happened between June and September 1998 that induced such a move of voters' positions towards FHC? Any straight answer to such a complex question is bound to be incomplete and superficial. However, one can hardly overlook the correlation between the international financial crisis[9], the shortening of Brazilian foreign reserves and the increasing support to FHC. The model presented here suggests an answer to this puzzle: as the campaign unfolded, voters became aware of the complex international situation Brazil was facing, which induced them to accept a lower level of social output (high unemployment) in order to reelect the President. In other words, FHC was successful in convincing voters that the state of the world was very unfavorable, so that the achievements of his government appeared reasonable, although very poor in terms of employment: voters became less demanding and FHC was reelected.

7 A discussion on the bargaining cost function

The results on vote splitting derived from the model rely strongly on the assumptions on the bargaining cost function $c(p, \pi)$. Although that function represents an attempt to capture the main characteristics of the bargaining process, it is exogenously postulated, which suggests an analysis on how such a bargaining could be endogenized. This section presents a brief account of the attempts that have been made in the literature in order to incorporate bargaining in models of electoral control, and discusses the challenges for the present model.

Models that explore reelection as the main electoral control mechanism typically do not incorporate the Legislature, as in Barro (1973) and Ferejohn (1986). However, bargaining plays a central role in models in which the Legislature is considered. There are basically two families of such models. The first family considers that the policy outcome is a consequence of an exogenously postulated bargaining process. The process may be as simple as a selection between four possible outcomes, as in Fiorina's studies (1988, 1992, 1996), or may be given by a differentiable function which depends on the proportion of opposition in Congress in a specific linear way, as in Alesina and Rosenthal's studies (1989, 1995, 1996), or in a more general way, as in the present model.

The second family derives the policy outcome as an equilibrium of a well defined bargaining game. The seminal article which started this literature is

[9] The Russian crisis exploded precisely on August 1998.

Baron and Ferejohn (1989). Baron and Ferejohn's model is based on the distributive approach to decision making in a Legislature: There are n representatives, each one of them trying to influence the budget in order to have more resources allocated to his own district[10]. A stochastic recognition rule selects a representative to make a budget proposal, which is them voted according to different institutional rules[11]. One of the main results is that the proposer has a strategic advantage, being able to direct a higher proportion of the budget to his district.

The basic Baron and Ferejohn study was concerned with the internal bargaining process in Legislatures and did not consider bargaining with the Executive. Subsequent articles extended that model to include such frictions. An interesting study in presented is Chari et al. (1997). That paper's conclusions regarding voters preferences are two-fold. On one hand voters in a district tend to prefer a liberal representative in the Legislature, in order to influence the budget resources towards that district. On the other hand, voters favor a conservative President in order to reduce overall spending and therefore compensate for the pork-barrel effect associated with the distributive approach to political decision making.

Another extension to bargaining with the Executive, based on a more partisan approach to political bargaining, was developed in Bugarin (1999). In that model parties have fixed ideological positions and the result of a clearly specified bargaining game suggests that voters split their ticket in order to avoid extreme policies in bad states of the world.

The present model does not belong to the basic distributive framework neither to the partisan approach to politics, which makes it a challenging task to incorporate bargaining. In order to introduce a bargaining game here it is necessary to answer a question on what motivates legislators' behavior with respect to a budget proposal by the President. Given that there are no distributive concerns, representatives are not moved by a pork-barrel type of argument. Moreover, ideology plays no role in the present model. It remains to consider the electoral connection: how does a representative's decision affect his parties' electoral success? The main difficulty here is to define what are the preferences of a party in the Legislature: does a party want to control the highest possible number of seats in the Legislature or does it give priority to controlling the Executive?

One way to deal with this issue is to assume that the Executive is the highest valued position in politics, so that a Legislator acts in order to maximize the probability that an incumbent from his party is elected President. In that case, the opposition party in the Legislature has an incentive to increase the cost of passing a budget to the Executive for a very simple reason: higher opposition may lead a political gridlock, which may reduce the popularity of the ruling party and, hence, increase the probability of victory of the oppo-

[10] For more details on the different theories of Congressional institutions, see Shepsley and Weingast (1995).
[11] Closed or open amendment rules, finite or infinite number of sessions.

sition party. Therefore, putting as much cost as possible on the bargaining process may be seen as a weakly dominant strategy for the opposition party.

The literature on the Congress points out many ways representatives use to increase the cost of passing a law in a Legislature, generically known as filibusters[12]. If one builds a model of stochastic recognition rule, in which the budget proposal is referred to each representative with the same probability, then it follows that the higher the representation of opposition in the Legislature, the costlier it is to the Executive incumbent to pass a budget proposal which is in accordance to a basic assumption on the role of opposition in this model. However, it is important to stress that the previous discussion is only intended as a guideline for future extensions of the present model, aiming at endogenizing the negotiation process between the Executive incumbent and the legislators. Indeed, a nontrivial effort is needed in order to build the details of the bargaining game. Such an extension is presented here as a suggestion for future research.

8 Conclusion: An agenda for future research

This article begins the construction of a unified framework for the analysis of both reelection and vote splitting as mechanisms of Executive incumbent control.

The findings are three-fold. First, if a last period does exist, it is always optimal for the voters to split their tickets in that period, in order to increase the cost to the incumbent of passing an expensive budget and, therefore, reduce the last period overspending. This result is robust in that it does not depend on the realized state of the world.

In the first period, however, voters' attitude towards vote splitting depends on the likelihood of being in a good state of nature. If the good state is likely enough, voters will also split their ticket in the first period, whereas if a bad state is more likely to occur, voters will prefer a unified government. Therefore, this model finds that, in equilibrium, voters are sensitive to the extra burden of negotiation with the Legislature in an already bad state and understand that electoral control will increase by reducing this burden; conversely, in a good state the corruption opportunities are higher and consequently it is important to reinforce opposition in the Legislature as a means of Executive corruption control.

Finally, voters' attitudes towards reelection are also determined by the likelihood of a good state. If a good state is more likely, voters will require a higher level of social output (n^*) for reelection, which will only be met when the realized state is indeed good. Therefore, when a bad state occurs the incumbent is not reelected, i.e., non-reelection occurs with positive probability. On the other hand, if a bad state is more likely, voters will require a lower

[12] See, for example, Oleszek (1996).

level of social output for reelection which will be met in both states of nature, so that reelection will always occur.

The model is an attempt at building a unified theory of political decision making in which voters, candidates and elected representatives are all strategic players. The results obtained are interesting and sensitive. However, the framework is still limited and can be extended in two main directions. First, an exploration on the robustness of the results obtained are in order: what happens when the issue space is multidimensional? Does the bigger number of parties change any of the results? How can the framework be extended to an infinite number of periods, if the executive incumbent is seen as a party, as in Ferejohn (1986)? What are the effects of term limits? What if the parties are not so totally identical? How should the cost function be endogenized? Those are all important issues that need to be dealt with in order to assess the generality of this paper's contribution.

Second, as was explained in Ferejohn (1986) and Bugarin (1999), a good theory of political decision making must involve different aspects of agents' preferences. Voters' utilities, for example, have a *sociotropic* content which, like here, values elected officials' performances, but also an *ideological* component which, like in Alesina and Rosenthal (1996), prefers certain types of policies rather than others. Elected officials, on the other hand, value office holding, like in this model and Ferejohn (1986) – their utilities' *pragmatic* component – but are also concerned about which type of policies are implemented – their utilities' *ideological* part. A model that involves both instruments of vote splitting and reelection in a framework where agents' utilities have sociotropic, pragmatic and ideological components is the ultimate objective of this line of research.

Appendix

Proof of Proposition 1

Given the assumption of an interior solution for the incumbent's maximization problem at period 2, the optimal overspending level $\hat{e}(r, \pi_2)$ is the solution e to the following first order condition:

$$\beta Br = (r+e)^2 \frac{\partial}{\partial p} c(r+e, \pi_2)$$

Suppose now that π_2 increases. If e remains constant, then $\frac{\partial}{\partial p} c(r+e, \pi_2)$ increases by condition (c-5), which is a contradiction. On the other hand, if e increases, keeping π_2 constant first, $\frac{\partial}{\partial p} c(r+e, \pi_2)$ increases by (c-2); then increasing π_2, $\frac{\partial}{\partial p} c(r+e, \pi_2)$ increases by (c-5), which is, again, a contradiction. Therefore $\hat{e}(r, \pi_2)$ is a decreasing function of π_2. Hence, the best choice for the voters at period 2 is $\pi_2 = 1$, i.e., vote splitting is optimal.

Proof of Proposition 2

Suppose that voters observe the state of nature is r. Then, the function $\tilde{e}(r,\pi)$ is the smaller solution e to the following problem:

$$v(r,e,\pi_1)+\delta v(r,\hat{e}(r,1),1)=v(r,\hat{e}(r,\pi_1),\pi_1)$$

Taking first order derivatives of that equation yields:

$$\frac{\partial v}{\partial \pi}(r,\tilde{e}(r,\pi_1),\pi_1)+\frac{\partial v}{\partial e}(r,\tilde{e}(r,\pi_1),\pi_1)\frac{\partial \tilde{e}}{\partial \pi_1}(r,\pi_1)$$
$$=\frac{\partial v}{\partial \pi}(r,\hat{e}(r,\pi_1),\pi_1)+\frac{\partial v}{\partial e}(r,\hat{e}(r,\pi_1),\pi_1)\frac{\partial \hat{e}}{\partial \pi_1}(r,\pi_1)$$

By definition of $\hat{e}$, $\dfrac{\partial v}{\partial e}(r,\hat{e}(r,\pi_1),\pi_1)=0$. Moreover, since $\tilde{e}(r,\pi_1)<\hat{e}(r,\pi_1)$, we have $K:=\dfrac{\partial v}{\partial e}(r,\tilde{e}(r,\pi_1),\pi_1)>0$.

Finally, since $\dfrac{\partial v}{\partial \pi}(r,e,\pi)=-\dfrac{\partial c}{\partial \pi}(r+c,\pi)$, the above expression yields:

$$\frac{\partial \tilde{e}}{\partial \pi_1}(r,\pi_1)=K^{-1}\left[\frac{\partial c}{\partial \pi_1}(r+\tilde{e}(r,\pi_1),\pi_1)-\frac{\partial c}{\partial \pi_1}(r+\hat{e}(r,\pi_1),\pi_1)\right]$$

Therefore, the sign of $\dfrac{\partial \tilde{e}}{\partial \pi_1}$ depends on whether $r=l$ or $r=h$. If $r=h$, then $p^*<h<h+\tilde{e}(h,\pi_1)<h+\hat{e}(h,\pi_1)$. Thus, by (c-4), $\dfrac{\partial c}{\partial \pi_1}(h+\tilde{e}(h,\pi_1),\pi_1)>\dfrac{\partial c}{\partial \pi_1}(h+\hat{e}(h,\pi_1),\pi_1)$, so that $\dfrac{\partial \tilde{e}}{\partial \pi_1}(h,\pi_1)>0$. In this case, $\pi_1=0$ is the optimal choice to the voters. Therefore, in a bad state of the world, voters prefer a unified government to a divided one.

Conversely, if $r=l$ and $\hat{e}(l,0)$ is small enough, then $l+\tilde{e}(l,\pi_1)<l+\hat{e}(l,\pi_1)<l+\hat{e}(l,0)<p^*$. Thus, by (c-4), $\dfrac{\partial c}{\partial \pi_1}(l+\tilde{e}(l,\pi_1),\pi_1)<\dfrac{\partial c}{\partial \pi_1}(l+\hat{e}(l,\pi_1),\pi_1)$, so that $\dfrac{\partial \tilde{e}}{\partial \pi_1}(l,\pi_1)<0$. In that second case, $\pi=1$ is optimal to the voters. Therefore, in a good state of the world, voters prefer divided government.

Proof of Proposition 3

Since $\tilde{e}(l,1)<\hat{e}(l,1)$ it follows that $\tilde{n}_l>\hat{n}_l$. Similarly, since $\tilde{e}(l,0)<\hat{e}(l,0)$ it follows that $\tilde{n}_h>\hat{n}_h$.

Moreover, since $l+\tilde{e}(l,1)<p^*<h+\tilde{e}(h,0)$ one concludes that $\tilde{n}_l>\tilde{n}_h$. Finally, since $l+\hat{e}(l,1)<p^*<h+\tilde{e}(h,0)$ one concludes that $\hat{n}_l>\tilde{n}_h$.

Note that the last two inequalities were derived from the assumption that $\hat{e}(l,0)$ is small enough compared to l, i.e., $l+\hat{e}(l,0)<p^*$, made in Proposition 2.

Comparing the above inequalities it follows that: $\hat{n}_h<\tilde{n}_h<\hat{n}_l<\tilde{n}_l$.

Proof of Proposition 4

Given that the only choices of n^* which have not been shown strictly dominated are $\tilde{n}_h$ and $\tilde{n}_l$, it is sufficient to compare voters' utilities for each one of those two choices.

If voters choose $n^* = \tilde{n}_h$ their expected utility is $V_h = \rho\varphi(\hat{n}_l) + (1-\rho)\varphi(\tilde{n}_h)$. On the other hand, if voters choose $n^* = \tilde{n}_l$ their expected utility is $V_l = \rho\varphi(\tilde{n}_l) + (1-\rho)\varphi(\hat{n}_h)$.

Now the proof follows from the fact that,

$$V_h > V_l \Leftrightarrow \rho < \frac{\varphi(\tilde{n}_h) - \varphi(\hat{n}_h)}{\varphi(\tilde{n}_l) - \varphi(\hat{n}_l) + \varphi(\tilde{n}_h) - \varphi(\hat{n}_h)}.$$

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