REVISING AGGREGATION AND UPDATING OF INPUT-OUTPUT MATRICES

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Abstract

This paper systematically surveys the theory and challenges to the aggregation and updating of input-output matrices. We are concerned with the static Leontief model; that is the matrix A of technological coefficients. Firstly, we deal with the analysis of unbiased aggregation and show that the necessary condition to be satisfied are rather severe and unlike to obtain in practice. Thus, the importance of looking for simpler criteria to solve practical problems. Secondly, we consider the bipropontional adjustment, mutation, for the updating of such matrices – the RAS method. This is only one of different ways to approach the problem. However, alternative methods can only perform as well or better along certain special contexts. We conclude that for aggregation and updating of input-output matrices there is a long and winding list of challenging questions.

Keywords: Aggregation; updating; Input-Output Matrices

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1. Introduction

From the early days of research on interindustry or intersectoral relationships, investigators have recognized the importance of the aggregation problem and the fact that the results of the research depend upon the particular procedures used to combine industries or sectors.[1] A large number of researchers have been concerned with both theoretical and practical aspects of this subject. On the theoretical side, the intersectoral relationship is part of a scheme which first formulation goes back to Walras and Marx.[2] On the empirical side, the construction of statistical tables so as to provide us with analogues of theoretical models starts with Leontief when he gave his theory empirical context and published the first input-output table for the USA.

While the aggregation problem has been widely recognized and much discussed in the literature it would hardly be an exaggeration to say that practical difficulties of collection, efficient utilization and interpretation of data necessary to build up simultaneously reliable and manageable input-output tables constitute some of the principal obstacles in the path of planning and forecasting models.

To give an idea of the magnitude of the task involved in the buildup of an input-output table it is sufficient to visualize that the representation of a modern economy by 100 industries or sectors is not very detailed. But if the sales of each industry to each other are arranged in rows and the purchases of each from all others in columns, we obtain a table 100 x 100 = 10,000 cell. If we represent an economy by 200 different industries, there would be 40,000 cells, a truly formidable number of possible interconnections (of course, there will be many zeros, because some industries have no dealings or direct connections with some other industries). Even on the smallest scale direct or intuitive interpretation could be made rather difficult on the basis of the figures above.

As we know, the usual assumptions of an input-output matrix include just one product from each industry, strictly fixed coefficients, complementarity among inputs, constant returns to scale and exactly known technical coefficients. Disregarding the practical impossibility of building up such a matrix it is clear that for the analysis of most problems conceived to be important in economics such detailed matrix would scarcely be useful. At the same time it is quite obvious that data collection of large matrices is both expensive and time consuming. So, much thought is given to the problem of obtaining a matrix that can be manageable and preserve as much as possible the properties of the ideal matrix.

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[1] Since we are only concerned with the aggregation in input-output models we need to point out firstly that aggregation is a very broad subject and by no means confined to Economics (see Fisher, 1969) and secondly some earlier studies on aggregation in economics as a general problem are by Thiel (1954), Nataf (1962) and Green (1964).

[2] As Morishima (1973, p.80) states "... Marx's view of aggregation is relatively clear, though not explicit".

One question that arises is whether physical data or monetary data should be used. Of course, items cannot be grouped together unless they have some sort of common denominator. Although physical units might constitute a common denominator for some goods, the only common denominator which enables much aggregation to be carried out is in monetary terms and most of the input-output tables in existence were built using this approach. However, it must not be assumed that this substitution of monetary for physical units avoids all the problems of nonadditivity. It is clear that different accounting systems, used by firms (different methods of depreciation, valuation of inventories, etc.) make the meaning of industry figures rather doubtful. So the study of the entire complex of accuracy or existing data is not only helpful but also indispensable in the understanding of existing tables and in the designing of programs for the collection of new, improved data. As Leontief (1960) states:

> "... the practical choice is not between aggregation and nonaggregation but rather between a higher and lower degree of aggregation." (p.208).

Nevertheless, there are many alternative ways of aggregating economic data and different classifications may lead us to different interpretations of the state of the economy. It is also true that the degree of aggregation to apply and the concepts to use depend on the problems which one wants to examine and the volume of reliable data. As most of the existing matrices are designed for "general" purposes it happens that given such a matrix and having a particular use in mind, some-times we need further aggregation.

So far we have been concentrating attention on the aggregation problem. Now, we intend to give some hints on the updating of Input-Output matrices. Surely, the long lag frequently encountered in the construction and publication of such tables in a basic constraint in multisectoral applications of this instrument in policy analysis. On the other hand, often, the use of outdated databases leading to inappropriate interpretation to policy questions is well known, This occurs particularly when there exists a significant structural change in the economy in the period being addressed.

Input-Output tables refer to a given period in time-specifically to the year to which the data is collected and this may become a severe trouble to the economic investigators. Of course, to produce, year after year, a new Input-Output matrix is beyond the capabilities of data collection. Surely such effort is both expensive and time-consuming. Consequently, already quite early in the tradition of input-output analysis, these have been studies devoted to using other information that new tables each year to generate the relevant statistical information.

The updating of Input-Output matrices of technical coefficients reflects technological change in a closed Leontief model produced by different sectoral growth rates, changes in the internal structure of the economy, variations in the price systems and/or changes in the final demand requirements. The first systematic formalization of such changes was introduced by Stone (1961) and Stone & Brown (1962). This objective was to devise a procedure that could be used to update a given Input-Output table without having to generate a completely new set of inter-industry data.

Some improvement in the approach is due to Stone (1963) as the “RAS Method”, which consists of interactive updating technical coefficient table by taking into account two different simultaneous effects: i) upward and downward trends in the degree of production of different industries or sectors (production effect) and ii) relative shifts in input requirements of particular industries or sectors (substitution effect). Subsequently, updating methods have been concentrated on optimization techniques and the development of corresponding solving algorithms. For instance, the works of Nijhamp & Paelinch (1974), Teixeira & Pacca (1977) and Teixeira & Silva (1978).

There has been a resurgence of interest in alternative updating adjustment methods recently. Most of the new techniques are a hind of reformulation of the RAS Method. It has been shown that the original RAS technique presents a number of the theoretically appealing properties and the new reformulations tend to deal with specific problems alien to what we consider relevant here. They can perform as well or better along certain dimensions and in certain contexts. However the original RAS approach is the fundamental one.

At this point we present a typical Input-Output structure in table 1 which is considered the starting framework for both on aggregation and updating techniques.

Table 1: INPUT-OUTPUT STRUCTURE

OUTPUT / INPUT		SECTORS				u	y	x
		1	2	…	n			
SECTORS	1	x_{11}	x_{12}	...	x_{1n}	u_1	y_1	x_1
	2	x_{21}	x_{22}	...	x_{2n}	u_n	y_2	x_2
	…	…	…	…	…	...	…	…
	n	x_{n1}	x_{n2}	…	x_{nn}	u_n	y_n	x_n
z		z_1	z_2	…	z_n	$\sum z_j=\sum u_i$		
v		v_1	v_2	…	v_n		$\sum v_j=\sum y_i$	
x		x_1	x_2	…	x_n			$\sum x_j=\sum x_i$

Notice that column x contains the sectoral gross output; line v indicates the added value (wage, profit, etc); column y shows the final (consumption, investment, etc); column u characterize the inflow of input from sector i to each sector; line z shows the input received by sector j from all sectors. Finally, x_{ij} with (i,j = 1, 2, 3, …, n) indicates the input from sector i to sector j during the period.

It is important to mention that we are dealing with a linear model where x_{ij}=a_{ij} x_j $\forall\, i, j$, where A={a_{ij}} is the matrix of technical coefficients and 0≤a_{ij}<1. Furthermore, in order to guarantee the productivity of an indecomposable economic system is required that the Leontief matrix (I-A) satisfies the Hawkins-Simon (1949) condition. On this matter see Morishima(1964). Notice that consumers preferences are often neglected in the Input-Output literature, which makes the long-run analysis of the stability of the technical coefficients essentially supply-oriented, as pointed out by Mello Jnr & Teixeira J. (1977).

In this paper, after this introduction to the literature on aggregation and updating of Input-Output matrices, in section 2 we deal with the exact aggregation problem. Section 3 examines a balance of gains and losses on aggregation of Input-Output matrix. Section 4 shows a systematic presentation of the "RAS Method" and extensions. Section 5 concludes.

2. The Exact Aggregation of an Input-Output Matrix

Let us suppose that we have an original input-output matrix and it is desirable to carry out (further) aggregation for a particular purpose in a way that would be certain to satisfy the theoretical criteria for avoiding bias. For convenience let us call any column of the aggregated system a sector. Let A be the original (n x n) matrix before the aggregation and $\bar{A} = [\bar{a}_{IJ}]$ the aggregated matrix where (I,J = 1,2,...,m). Let x be the (n x 1) vector that gives the gross output before the aggregation and $\bar{x}$ the vector after the aggregation, now (m x 1). If we call y the (n x 1) final demand vector before the aggregation and $\bar{y}$ the (m x 1) after, it seems natural to assess the quality of the aggregation through the requirement that the results obtained from the original matrix and the aggregated must coincide with each other. In order to see this condition satisfied let us call Z an (m x n) aggregational operator. This aggregator is a matrix which j^{th} row consists of i zeros followed by (j – i) units and (m – j) zeros, where $z_{ij} = 1$ if and only if, j is to be included in the I^{th} aggregated sector. That is:

$$Z = \begin{pmatrix} 1 \dots 1 & 0 \dots 0 \dots 0 \dots 0 \dots 0 \\ 0 \dots 0 & 1 \dots 1 \dots 1 \dots 0 \dots 0 \\ \dots \dots \dots & \dots \dots \dots \dots \dots \dots \dots \dots \\ 0 \dots 0 & 0 \dots 0 \dots 0 \dots 1 \dots 1 \end{pmatrix}$$

The Z matrix above defined leads us to simple aggregation, i.e., aggregation as a simple sum with unit weights attached to each original sector. Problems of weighted aggregation are studied by Morimoto (1971) and Hatanaka (1952). Simple aggregation is a special case of weighted aggregation and it can be shown that most of the fundamental relationships and hence the theorems established for the case of simple aggregation hold equally in the case of weighted aggregation, by simple changing the weights of each original industry from unit to some given positive value and re-interpreting the conditions of the theorems. Nevertheless, it should be noted that in the case of weighted aggregation new important problems emerge, and here we are only concerned with simple aggregation.

In order to continue our research for exact aggregation let us define I as the (n x n) identity matrix associated with the original matrix and $\bar{I}$ as the (m x m) identity matrix associated with the aggregated matrix. Defining the matrices and vectors, where obviously m<n, we may write:

$$y = (I - A)x \tag{2.1}$$
$$\bar{y} = (\bar{I} - \bar{A})\bar{x} \tag{2.2}$$

If the aggregation causes no error we will have:

$$Zy = \bar{y} \tag{2.3}$$
$$Zx = \bar{x} \tag{2.4}$$

Substituting (2.1.) and (2.2.) in (2.3.) we get:

$$Z(I - A)x = (\bar{I} - \bar{A})\bar{x} \tag{2.5}$$

$$Zx - ZAx = \bar{x} - \bar{A}x \tag{2.6}$$

Substituting (2.4.) in (2.6.) we get as the condition for unbiased aggregation:

$$ZAx = \bar{A}Zx \tag{2.7}$$

This is to be true for any x. This implies that $ZA = \bar{A}Z$.

The conditions arrived at are severe. There is little probability that they will be fulfilled. As Kossov (1972) states:

> "From the economic point of view this stipulation... means that the aggregation will yield satisfactory results only when a chance in the production pattern within the consolidated group of sectors does not influence the aggregated coefficients." (p.242.)

From the practical standpoint it is clear that the condition is very severe and extremely unlikely to occur in any real economic context. Nonetheless, the approach discussed can help to solve the problem and stress that the possible bias is a function of the level of aggregation and the criteria used.

It was one purpose of this section to point out the severe restrictions which would make a consolidation perfectly safe. The other purpose is to indicate that the practical solution consists in the acceptance of an approximation solution for this aggregation problem. Practitioners of this art have developed various theorems, criteria and ingenious ways in which a set of input-output data can be collapsed, with little appreciable error or, in Thiel and Uribe’s (1967) term, "loss of information".

A large number of criteria have been proposed for approximate aggregation. Among them we have: similarity of coefficients, partial aggregation, proportionality of final demand, uncorrelated final demand, minimal distance idea, similarity of demand patterns, and the capital intensity of the activities. There are often formidable difficulties in applying these criteria for general consistent aggregation and normally several groupings need to be made if the original number of industries is large, or if input structures of members in the same group are not the same in all details.

If aggregation is justifiable in an absolute sense only spurious information is lost in the process. In a relative sense we can also justify it by showing that the real information lost is small and worth sacrificing because of positive advantages which aggregation brings.

We do not intend to put forward the above mentioned procedures of approximate aggregation, since the literature on this matter is well known. In the next section we only intend to show the balancing of gains and losses that occur when we do an aggregation.

3. Balance of Gains and Losses on Aggregation

One of the most crucial assumptions of the input-output analysis is that input proportions are fixed. If this assumption is justified or not depends on several other conditions being satisfied. Let us analyze some of these conditions. Firstly, relative input price changes cause substitution of one input for another or a sub set for another. This means that either price changes must be sufficiently small for there to be little substitution or the relative proportions of different inputs are fixed by technological considerations. A very fine classification of sectors will result in closely substitutable produced inputs being put into different sectors and the likely result will be that of price changes causing the output of one sector to be substituted for that of another in the input combination of a third. In this case a broader aggregation is likely to result in dose substitutes being grouped into one sector, so that there would be less chance of significant substitution of the produced inputs of various sectors.

Secondly, it must be assumed that there is no significant excess capacity within any industry. With excess capacity, or very large inventories of certain inputs, it may be possible to increase the output without proportional increases in all inputs. A great degree of aggregation may indicate that excess stocks of inputs by some sectors would tend to be cancelled out by depleted stocks in other sectors.

Thirdly, a great degree of aggregation will tend to cancel out errors introduced by indivisibilities.

Fourthly, it is possible that, with a high degree of aggregation changes in individual industry coefficients will balance out over a whole sector, thus some industries become more capital using and others less so. It is difficult to place too much reliance on the prospect of averaging.

Fifthly, it must be considered that depending on the degree of aggregation each sectorial classification will cover a range of different products. Either we should assume that each product within the sector classification has the same input structure, or that an expansion of the sector results in an equi-proportional increase in all products within the classification. In this case, the degree of aggregation is a two edged sword: on one hand, a very fine sectorial classification would tend to guarantee a homogeneous input structure. On the other hand, greater aggregation again would allow for increased possibilities of the cancellation of distorting effects.

Another point that deserves comment is the aggregation of production of different time lags and the emergency of new industries in a growing economy. The second point presents no great conceptual or practical difficulty for it can be handled relatively easily within an existing input-output framework. As new industries emerge and commence engaging in intermediate material and capital transaction with other sectors of the economy, we merely fill in the so called "empty boxes" of our input-output table by adding another row of intermediate and final outputs and another column of material and factor inputs to represent the activities of this new industry. We could then compute the relevant technical coefficients of production on the basis alternative approaches. For instance, on the basis of the statistical experience of similar industries already established in countries of comparable economic development. Chakravarty (1968) studied the problem. It is clear that the greater is the degree of aggregation the less important is the problem of creation of new industries.

A point that deserves some concern is that of the lag problem. We cannot say too much about it. This is, in fact, an almost insoluble issue because different industries or sectors probably have different time lag and in the aggregation we disregard those differences. Eckaus and Parikh (1968) had experimented with a rather sophisticated gestation lag in their “Target” and “Transit” models[3]. It seems that we can only “solve” the problem through the updating of the technological coefficients on the Leontief model. This is the theme of next section.

4. The Updating of Input-Output Matrices Revisited

The first systematic presentation of technical change in the context of input-output tables was made by Stone (1963) in what he called the "RAS-Method". It consists of an attempt at updating the input-output matrices taking into consideration simultaneously two effects. They are:

(a) Relative shifts in the required input proportions of certain industries; and

(b) The changes in productivity; i.e., upward and downward tendencies in an industries degree of fabrication.

The first is called "substitution effect" which requires a adaptation of the rows. The second "fabrication or productivity effect" requires a systematic adaptation of the columns of the input matrix A.

The "RAS-Method" is also referred to as the "Biproportional Method". This new teminology was introduced by Bacharach (1970) and does not constitute an attempt to substitute names but to help to abstract the mathematical characteristics from economic associations. In fact the method is rather general and has been used outside the inter-industry output applications. We will, however, use only Stone's terminology. "RAS" is a code name that comes from the notation:

$$a_{ij} = r_i a_{ij} s_j \quad \text{with} \quad i, j = 1, 2, \dots, n \qquad (4.1)$$

where a_{ij} and a_{ij}^* are respectively the values of the (i, j) input-output coefficients at the initial (or basic) period and the target period. Notice that r_i and s_j are two types of multipliers, the first is the substitution effects and the second is the fabrication one.

Turning to the matrix notation, we say that the adjustment operation, in order to obtain the new A* matrix from the basic A matrix, consists in the premultiplication of A by

[3] They experimented a dynamic structure with 3 years gestation lags, with individual time periods each one year in length. Subsequently they adopted for the other models the simplifying assumption that the individual time periods and gestation lags were of identical length, there years.

a diagonal matrix $\hat{r}$, and the simultaneous post-multiplication by a diagonal matrix $\hat{s}$. Thus, the relation between the basic (A matrix) and the new matrix (A*) is given by:

$$A^* = \hat{r}\, A\, \hat{s} \tag{4.2}$$

Through the premultiplication the adjustment of the rows is obtained and through the post-multiplication the column's adjustment is obtained, provided that $\hat{r}$ and $\hat{s}$ are known. In essence the problem consists of finding a matrix having prescribed rows and columns and the procedure only makes sense if substitution and fabrication effects exert a systematic uniform influence upon the rows and columns of the input-output table through time.

In order to apply the method, the following data is required: vector x of sectoral gross output, vector v of primary input per sector and vector y of final demand. Using these data the vector u of total intermediate outputs and the vector z of total intermediate inputs can be calculated through the following expressions:

$$u = x - y \tag{4.3}$$

$$z = x - v \tag{4.4}$$

In order to proceed we need the closed Leontief model and the balance equation for production value plus factor costs. They are respectively:

$$x = A^*x + y \tag{4.5}$$

$$x = \hat{x}\,(A^*)'e + v \tag{4.6}$$

Where $\hat{x}$ is a diagonalized x vector, e is the unit vector and the comma indicates a transposed matrix.

From (4.5) we obtain:

$$x - y = A^*x \tag{4.7}$$

On the other hand, with (4.3):

$$u = A^*x \tag{4.8}$$

From (4.6) we have:

$$x - v = \hat{x}(A^*)'e \tag{4.9}$$

Therefore :

$$z = \hat{x}(A^*)'e \tag{4.10}$$

Substituting (4.2) into (4.8) we get:

$$u = \hat{r}\, A\, \hat{s}\, x \qquad (4.11)$$

Replacing (4.2) into (4.10) yields:

$$z = \hat{x}(\hat{r}\, A\, \hat{s})' e \qquad (4.12)$$

Thus:

$$z = \hat{x}\hat{s}A'\hat{r}e \qquad (4.13)$$

But $\hat{r}\, e = r$ where r is a vector containing the diagonal elements of $\hat{r}$.

Therefore:

$$z = \hat{x}\hat{s}A'\hat{r} \qquad (4.14)$$

The systems (4.11) and (4.14) according to Nijkamp and Paelinck (1974):

> "... are a set of nonlinear equations containing the unknown elements of r and Since the number of equations is equal to the number of unknown elements, this system can, in principle, be solved." (p.3.)

The solution is obtained using the following iterative method: Firstly, we insert into (4.11) the identity matrix as an initial solution for $\hat{s}$ and then to solve for the resulting value $\hat{r}$. This value of $\hat{r}$ is then substituted into (4.14) and thus a new value of $\hat{s}$ is obtained. This value is again substituted in (4.11) and a new value $\hat{r}$ is obtained. This procedure is repeated until a solution for $\hat{r}$ and $\hat{s}$ is obtained with the required accuracy.

Bacharach (1965) studied the mathematical properties of the method and he shows that the convergence and uniqueness of the solution is assured[4]. Nevertheless, it is clear that the "RAS-Method" of updating a given input-output matrix exhibits obvious computational difficulties and requires a considerable volume of data. It is possible to go to a second stages in order to test the stability of the projected input-output coefficients, and also to obtain through interpolation input-output matrix for the periods in between the initial and the target period. When we compare two updated rnatrices obtained by using the "RAS-Method" or other approach, we may expect some differences in the patterns of the coefficients.

Statistical information for many countries reveals that in the past several types of technical change occurred and some of these will certainly occur in the future. The first rows concerns substitution among energy resources, especially shifts from wood to coal, oil, gas, or electricity atomic. The second type of technical change is the substitution from

[4] Mello Jnr. and Teixeira J. (1993) show an alternative proof to the existence of a solution to the RAS updating technique.

natural raw materials to manufactured goods, as in the case of chemical or synthetic fibres which have largely replaced cotton. Changes in product-mix also tend to move in favour of more manufactured goods. The third change relates to the general increase in the use of manufactured and service inputs. The typical example here can be found in agriculture, where increased use of fuels, machinery parts, repairs and other services follow in the wake of a degree of mechanization. The last technical change that we would like to comment on is concerned with columns of input-output coefficients, while the previous changes are related to rows of the coefficient matrix. As Watanabe and Shishido (1970) state:

> "Over the long run, input-output coefficients for manufacturing industries tend to decline due to improvements in efficiency and fabrication particularly in chemical and machinery industries. The opposite tendency is found in the construction industry where prefabricated materials become more important and thus an increase of the column sum of these input-output coefficients is identified." (p.13)

The "RAS-Method" surveyed in this section is only one of many different ways of updating an input-output table. Other interesting alternatives are the "Linear Programming Method", developed by Matuszewski et al. (1964), and the "Quadratic Programming Method", studied by Nijkamp and Paelinck (1974).

After considering all points above, we think that it is worthwhile to discuss the construction of the A^* matrix by using a different approach from that of input-output estimates. The alternatives include engineering sources, forecasts by experts via .the "Delphi Method", the "Battelle-Columbus Technique", and less sophisticated forms of intuitive forecasting[5].

In its original restricted form, the Delphi technique was intended to replace direct debate among "experts" by a carefully designed programme of sequential interrogations. In it, sequentially derived principally from the fact that the respondents do not always agree in the initial cycle and, since consensus among them is of central interest, the experimenters have to feedback, in statistical form, information on responses from one interaction to the next. The process continues until a certain degree of agreement is achieved (this method is known as convergence of opinions in graph).

It should be noted that while consensus is of central interest, care is taken for it to be genuine and not to derive it artificially. Information exchanged through the means of questionnaires is fully exploited to this end. Furthermore, all efforts need to be made to ensure that anonymity of responses is preserved throughout this exchange.

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[5] For an interesting survey on forecasting techniques see R. Ayres: "Technological Rorecasting and Long Range planning", McGraw-Hill, 1969. For the Battelle-Columbus Technique see Fisher and Chilton (1972).

The intention is to eliminate the influence of coercion, unwillingness to abandon publicly expressed views, and the bandwagon effect of majority opinion.

We do not think that the Delphi technique is only useful for the exploration of the future, since an improved understanding of the past and present can also be attained. In this vein it is necessary to experiment with new techniques such as the Battelle Columbus Method, which involves the direct generation of A^* from technological forecasts. We call these techniques an ex-ante approach. The use of these techniques in the context of input-output analysis is relatively well known. Experts tend to use the Battelle-Columbus technique for the reasons given below.

In the Delphi method there is a play back and forth between a panel of experts and the people responsible for the research. So that, after the first set of questionnaires is obtained the numerical answers are assembled as distributions, stated in terms of means and quartiles, plus any pertinent comments by the experts. Unlike the Delphi method, the Battelle-Columbus Technique tries to concentrate only on a few experts for each sector (conventionally two) and provides each expert with one set of coefficients based on the knowledge already in existence. It is also usual to let the interviewer provide for continuing interaction between the expert and his earlier statements, as well as benchmark data, other expert and also with the background knowledge possessed by the interviewer.

An important problems for the use of this technique is that, one may argue , the complexity of the economy ensures that few persons know who ultimately purchases and uses a given sector's output, while many experts know what their sectors purchase as input. Anyway, it seems very useful to start the updating of a given A matrix alternative throughout the RAS method and then, by means of the Delphi or Battelle Columbus technique, to refine the original result. Teixeira (1975) did this with some empirical depth using Brazilian data.

5. Concluding Remarks

There has been a recent resurgence of interest in the aggregation problem of Input-Output matrices as well as in the updating of such matrices. In the present essay theoretical and empirical considerations as these two problems were revisited. Firstly, we considered the case of aggregation in the static Leontief model. Secondly, we dealt with the biproportional adjustment technique, called the RAS Method in the Input-Output literature, in order to update technological coefficients (the A* instead of the A matrix).

In order to treat the aggregation in the framework of the static Leontief Model, initially, we worked out the case of exact approach which would make a consolidation perfectly safe. As we have indicated, such treatment requires severe restrictions which we consider an heroic hypothesis. Being this the case, we need practical solutions based upon the acceptance of approximate solutions. Of course, we need to take into account a balance of gains and losses based on approximate solutions. This was considered in the present paper.

As we know the static model is based on the assumption that only current flows of inputs and outputs are relevant. Specifically, it is assumed that investments can be included as part of the final demand for a given good of a particular industry (sector) and that its magnitude in any given period is unassociated with the level of economic activity in that industry (sector). By so doing, the static Input-Output model divorces investment decisions from output objectives and capacity considerations.

By means of refinement of the static Leontief model we can realize that it is required to reconsider the traditional static Input-Output matrix in order to capture the dynamics of the economy. In this case the aggregation problem becomes even more dramatic. Here we did not include the aggregation problem in the context of dynamic Leontief models and extensions. It is our view that the theory necessary to deal with this case is not well developed in the literature.

Concerning the updating of technical coefficients of the Input-Output matrix, which we dealt with in this survey, our major interest was to consider the biproportional technique, commonly called the RAS Method, developed originally by Stone (1963.). As we have shown this approach has a member of theoretically appealing properties. The RAS method is a good alternative in comparison with various other approaches of examine comparative performances and examining the attributes of others adjustment methods.

Furthermore, the mathematical and computational technique is rather simple and, more important, the practical results, so far, tend to be competitive with those obtained through alternative approaches. In this vein, the RAS method continues to be commonly used and its choice is often a satisfactory decision. However, as it is well known, the dynamics of structural dynamics and change is a very complex subject, with important ramifications concerning its relationships with technological change and with inter-sectorial input-output coefficients.

Being this the case, the reader will come to understand how difficult is to offer a safe road to both the aggregation and updating of the coefficients of a Leontief framework. The conclusion on this theoretical and empirical literature is that we need to take into account roads not to be taken. In the last few couple of decades, a number of new approaches have emerged incorporating new algorithms, mathematical techniques and computational support, but they have not been particularly successful as a theoretical framework. It is not too difficult to fully appreciate just how long and winding the road ahead is in order to solve the aggregation and updating of Input-Output matrices. However, this requires another survey, in progress.

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